# Chapter 10: Advanced Option Trading Strategies

Reading this chapter you will be introduced to the following key concepts:

- Option sensitivities – the “Greeks”: delta, gamma, theta, and vega
- Revisiting the static option trading strategies: exposure to the Greeks with straddles and with the various types of spread trades
- Dynamic trading strategies with options: delta hedging, long/short gamma strategies

## Introduction

Because of their special properties, specifically the asymmetric payoff, options allow investors to implement trading strategies which cannot be implemented with only the underlying asset or with futures contracts on the asset. In a previous chapter we saw a number of these strategies and here we will return to the most important ones. Moreover, because the option price depends on not only the actual value of the underlying but also e.g. the volatility this type of derivatives allows investors to speculate in additional factors. Thus, option traders are often said to buy or sell volatility. In fact they could be said to buy and sell time also!

To analyse option positions traders often use a number of sensitivity measures. These are collectively referred to as the Greeks and each of these numbers measure the sensitivity of the option price to one of the factors affecting the option price. In this document we will discuss the most important of the Greeks in detail. We will also show how each position in options can be interpreted in terms of the exposure to each of these risk factors. This is particularly the case for the option trading strategies we have seen previously.

However, when considering options it is important to mention also that in real life traders do not just set up option portfolios and leave them to expire. First of all the trader may wish to hedge against some of the exposures and this could require rebalancing. Secondly, there is often a possibility of locking in the profit due to changes in any of these underlying factors as time goes by. This is done by trading actively the underlying asset. Finally, there is the possibility of trading of the options at any time prior to expiration to make profits in some cases. All of these cases which involve dynamic trading will also be discussed in this chapter.

Before getting into the details of trading strategies and exposure to different risk factors we will start by introducing the option markets and briefly review how one trades options. This chapter also includes an appendix which details how the Greeks can be calculate from the Black-Scholes formula.

## Option trading at exchanges

Options may be traded both on exchanges and over the counter, i.e. OTC. In the latter case, the OTC trading, investors potentially face the risk that the counterparty fails to pay. However, unlike with e.g. futures positions in options may not be exposed to this risk. The reason is that it is only for long positions in options that future payments will be received. A short position on the other hand is never exposed to counterparty risk as all payments from that party are received at the time the option is shorted. When trading on exchanges the counterparty risk is avoided since all trades happen with the option clearing corporation at the exchange.

For options trading at exchanges it again holds that these are completely standardized products. To make sure that there is always a sufficient amount of liquidity most exchanges use market makers to facilitate options trading. As always the market maker continuously quotes both bid and ask prices, or provides these when requested, without knowing whether the individual requesting the quotes wants to buy or sell options.

As it is the case with futures contracts margins are required when options are sold. Because the downside on a long options position is zero, no margin is required with this position. When a naked option is written on the other hand the margin is the greater of:

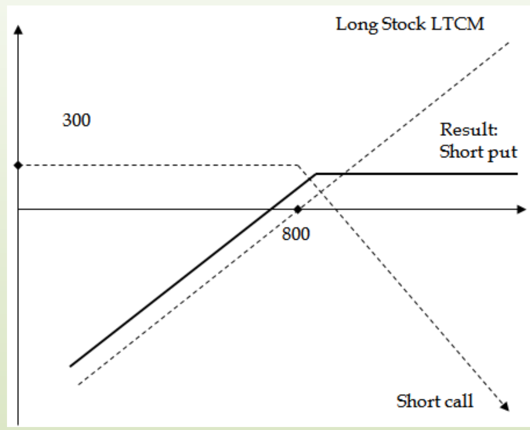
- A total of 100% of the proceeds of the sale plus 20% of the underlying share price less the amount (if any) by which the option is out of the money
- A total of 100% of the proceeds of the sale plus 10% of the underlying share price

For other trading strategies there are special rules which may vary from exchange to exchange and even from product to product.

Although prices may be quoted in dollar and alternative to this is in terms of implied volatilities. That is, the volatility needed in e.g. the Black-Scholes-Merton model to obtain a certain price. In

An example of an OTC option trade that went bad:

This by now well know deal was between the hedge fund called Long Term Capital Management and the Swiss bank UBS. This was a structured deal were UBS simultaneously sold a call option on 1 million LTCM stocks and bought 1 million stocks currently valued at $800 million. For USB this was the only way they could get a part of LTCM. The combined exposure is shown in the following figure:




That is, through this transaction UBS essentially sold a put option to LTCM where the underlying was the performance of the company itself. The result turned out to be a disaster for UBS who lost $682 when the LTMC hedge fund went down in 1998.

fact this is oftentimes the preferred way to quote options amongst traders. Because of the one to one correspondence of the price and volatility in the Black-Scholes-Merton model such a price conveys the same information.

## Option prices and price sensitivities

Options are complex financial instruments whose value depends on multiple factors. Specifically, the value of a plain vanilla option is affected by the following six factors:

1. The current stock price, $S_0$
2. The strike price, K
3. The time to expiration, T
4. The volatility of the underlying asset, σ
5. The risk-free interest rate, r
6. The dividends expected during the life of the option, q

For each of these factors Table 1 illustrates how the option price changes when either of these changes holding the other factors constant.

**Table 1: Summary of the effect on the price of a stock option of increasing one variable while keeping all other fixed. In the table a "+" means that when this factor increase the relevant option price increase, a "-" means that the option price decrease, a "?" means that the effect on the European option price may actually in some cases go in either direction.**[1]

| Variable | European call | European put | American call | American put |
|---|---|---|---|---|
| $S_0$ | + | - | + | - |
| K | - | + | - | + |
| T | +(?) | +(?) | + | + |
| σ | + | + | + | + |
| r | + | - | + | - |
| q | - | + | - | + |

The table shows that in general option prices are affected in the same direction whether these are American style or European style. The table also shows that in general put and call options are affected inversely by the factors with the exception being for changes in volatility. Volatility is special because it increases the spread of future asset prices. However, since options are "asymmetric" in their payoff, only the increased upside potential is important. Hence prices for put as well as call options increase when volatility increase.

Considering the complexity of the option pricing problem it should come as no surprise that in general it is very difficult to calculate an exact value of the product. In fact, in order to do this a

[1] For a detailed explanation and intuition about the effect of all these factors see e.g. *Options, Futures, and other Derivatives* by John C. Hull.

complicated mathematical model is often needed. However, without making any assumptions we can quantify the option value. The relationship is illustrated in Figure 1 below.

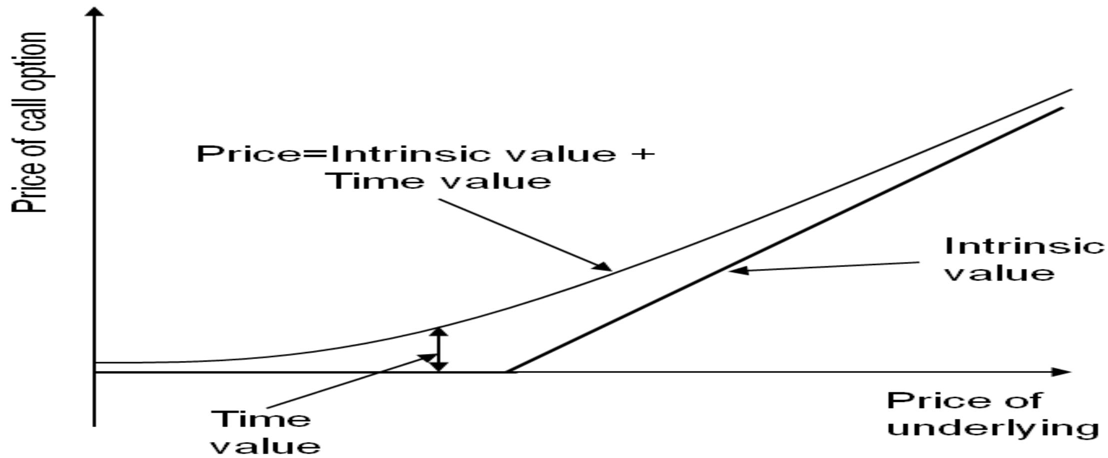



**Figure 1: The price of a call option as a function of the value of the underlying. The figure shows that the price is the sum of the intrinsic value, the value of exercising the option immediately, and the time value, the value associated with the possibility, however remote, that the option may increase in value due to volatility in the underlying asset.**

## Option sensitivities – the Greeks

Although each of the six factors in Table 1 affects the option price not all of them are equally interesting from a trading perspective. For instance, the strike price, K, is specified in the options contract and hence does not change through the life of the option. Thus it is not a factor which can be used for "trading". Also, the interest rate, r, can often be assumed to be constant over the life of the option, particularly if the option is of short maturity. And finally, the dividend yield can, without much loss of generality, be assumed to be constant also. This leaves the price of the underlying, $S_0$, the time to maturity, T, and the volatility, σ, as interesting factors when it comes to trading options. Note that the last two of these factors cannot be traded by using the underlying alone or say futures contracts on it, and is particular to options.

### The delta, Δ, of an option

There is little doubt that the most important factor affecting option prices is changes in the underlying asset. After all, it is from this asset that the option derives its value. The sensitivity of the option price to changes in the asset value is called "delta", denominated by the Greek symbol Δ. Formally, delta of a call option with price equal to C is defined as

$$\Delta_C = \frac{\partial C}{\partial S}.$$

To calculate this explicitly, a formula is needed for the option price like e.g. the Black-Scholes-Merton formula.[2]

However, even without a formula we can still quantify the delta of the option. This is easily done by looking at Figure 1 which shows a graph of the call price. From this it is clear that for very low values of the underlying the call option price does not change much when the underlying changes. Hence, delta is positive and close to zero for deep out of the money options. On the other hand, for high values of the underlying the option value increases virtually one for one when the underlying increases in value. Hence, delta is close to one for deep in the money call options. Between these two extremes the delta increases monotonically, and oftentimes it is found to be approximately equal to ½ for at the money options.

What about the put options? Well we could use the equivalent figure for the price or simply use the put-call parity to convince ourselves that $\Delta_P = \Delta_C - 1$. This follows by differentiating both sides of the parity and considering the sum of the elements on either side of the equality sign. Since the parity holds irrespective of the prices, the sum on either side of the equality sign should be the same. Moreover, because K does not change with S this term has a "delta" equal to zero, and the expression above is obtained.

The delta of an option also changes with time to maturity. In particular, for out of the money options the delta becomes closer to zero when the option is close to maturity. That is, for call options the delta, which is positive, decreases and for put options the delta, which is negative, increase. For the in the money options the opposite is found. At maturity the delta of a call option is in fact 0 for out of the money options and 1 for in the money options, and it changes in a discrete jump exactly when the option is at the money.

### The gamma, Γ, of an option

Although delta is a good approximation to the changes for small variations, it is only an approximation. In particular, as the underlying asset changes so does delta as we have seen. Moreover, when the option is at the money the delta may change quite quickly. The sensitivity of the option delta to changes in the asset value is called "gamma", denominated by the Greek symbol Γ. Formally, gamma of a call option with delta equal to $\Delta_C$ is defined as

$$\Gamma_C = \frac{\partial \Delta_C}{\partial S}.$$

Figure 1 may be used to quantify the gamma of an option. From this it is possible to see that for deep out of the money or in the money options gamma will be close to zero. On the other hand, gamma is maximized when the option is at the money. Moreover, as the curvature of the option price increases when the option arrives at maturity, gamma is larger for shorter term options.

[2] Note that the delta of the underlying, $\Delta_S$, is trivially equal to 1! The delta for a futures contract can be calculated from the spot futures parity.

What about the put options? Well we could use the equivalent figure for the price or use the put-call parity to convince ourselves that the gamma of a put and call option with the same strike price are the same. This follows immediately by differentiating on both sides of $\Delta_P = \Delta_C - 1$, since 1 does not change with S.

### The theta, Θ, of an option

The value of an option is clearly related to the time to maturity since this determines the time value of the option. As the maturity decreases this value decreases. The sensitivity of the option price to the passage of time is called "theta", denominated by the Greek symbol Θ. Formally, theta of a call option is defined as

$$\Theta_C = -\frac{\partial C}{\partial T},$$

where T is the time to maturity of the option. Thus, theta is always negative for a long position in options since time value increases with maturity T.

Although it is not possible to use the put-call parity to quantify the relationship for theta, in general, it can be said that theta is slightly more negative for call options than for put options with equal strike price. This on the other hand can be seen from the put-call parity.

### The vega of an option

The last of the interesting variables affecting the price of an option is the volatility, σ, which increases the value of any option irrespective of type or style. The sensitivity of the option price to a change in the volatility of the underlying asset is called vega.[3] Formally, vega of a call option is defined as

$$\text{Vega}_C = \frac{\partial C}{\partial \sigma}.$$

Note that vega is always positive for options irrespective of their type or style. Moreover, using the put-call parity it is seen that vega is the same for put and call options with equal strike price. Again this is easily seen by differentiating both sides of the parity with respect to σ and noticing that neither S nor K changes with this.

### Summarizing the option sensitivities

In the previous sections we have discussed the delta, gamma, theta, and vega of a simple call option. Performing the same analysis for the put option is easy and can often be done using the put-call parity as we have seen. Also, it is easy to see that if the option position had instead been a short position the "signs" of the sensitivities are inverted. This holds for all the sensitivities. Thus, we may summarize the sensitivities for any potential position in options as well as in the underlying as in Table 2. Note that due to the spot-futures parity futures contracts have the same sensitivity as the underlying.

[3] Note that vega is in fact not a Greek letter!

**Table 2: This table shows the sensitivities of various positions in the underlying and options to the most common "Greeks". Long means that if the factor increases the position will increase in value also, i.e. that there is positive dependency.**

| Position | Delta exposure (Directional) | Gamma exposure (Curvature) | Theta exposure (Time) | Vega exposure (Volatility) |
|---|---|---|---|---|
| Long underlying | Long | 0 | 0 | 0 |
| Short underlying | Short | 0 | 0 | 0 |
| Long put | Short | Long | Short | Long |
| Short put | Long | Short | Long | Short |
| Long call | Long | Long | Short | Long |
| Short call | Short | Short | Long | Short |

It is important to keep in mind that whenever investing in options an investor has exposure to each and all of these variables or "risks".

## Static trading strategies with options

While trading futures is like trading the underlying, it should be clear that trading options is quite different. In particular, in the previous section we saw that the price of an option is influenced by a number of factors as Table 1 shows. This means that when you have a position involving options you have exposure to each and every one of these factors. This, on the other hand means that by using options these factors can be "traded".

However, not all of these factors are equally interesting from a trading perspective. In fact the most interesting factors are the ones related to the Greeks, i.e. the option price sensitivities discussed above. In the next section we will discuss the issue of trading the Greeks in more detail. Such trades in principle only involve taking a position in one option. However, we have seen in a previous chapter that options are often combined to provide interesting payoffs, something which is possible because of the non-linearity. In the second section we revisit the combinations and the spreads in terms of exposure to the Greeks.

### Directional trading strategies – trading the Greeks

Table 2 provides us with the exposure of various positions in options to changes in the value of the underlying, the time to maturity, and the volatility in terms of the Greeks. As all option positions involve exposure to these risk factors the value of your position changes, should these factors change. As an example, consider a position which is long delta. To make money with this position you would want the underlying to increase in price while everything else remains the same. Thus, with options it becomes possible to trade on each of these factors by taking the appropriate position.

To find the appropriate position we simply "invert" the table. The result is shown in Table 3.

**Table 3: This table shows the profitable positions in terms of options Greeks for different expected market configurations. The left hand columns list different types of investor expectations about the future market. The right hand columns indicate the exposure investors should have to implement profitable strategies.**

| If you expect: | Your exposure should be: |
|---|---|
| The underlying to increase in price<br>The underlying to decrease in price | Long on delta<br>Short on delta |
| The underlying to move quickly<br>To move slowly or not move at all | Long on gamma<br>Short on gamma |
| Time to "pass by"<br>Time to "stand still" | Long on theta<br>Short on theta |
| Volatility to increase<br>Volatility to decrease | Long on vega<br>Short on vega |

While some of these trading strategies are standard it is worth noting that the option markets actually allow investors to buy time. I.e. you can be long time by shorting options as these have negative theta. What this means is that if nothing else happens you stand to make money. This is exactly what option traders hope for when taking the short end of the option and collecting the option premium. Moreover, the option markets allow investors to invest in volatility. That is, by buying options, and hereby obtaining a long exposure to volatility, an investor can construct a portfolio which increases in value if the market volatility increases.

There is one caveat though: when buying an option you are buying exposure to all these factors. Thus, in particular it is very difficult to get exposure to volatility without also getting negative exposure to time. That is, the portfolio which is profitable if volatility increases often loses money over time! But this is, of course, just another way of saying that nothing comes for free in financial markets.

## Revisiting the classical option trading strategies

In the following we will discuss a number of trading strategies which involves positions in options alone in terms of the exposure to the Greeks. Specifically, we will examine the straddle, the bull spread, the calendar spread, as well as the diagonal spreads. By looking at the exposure to the various Greeks we can easily discuss when such strategies may be beneficial. This last point is obviously related to the investor's anticipation of future market behaviour, and therefore the Greeks can be used to decide when to put such strategies in place.

### Straddle positions and the Greeks

A straddle is a position which is long a call and long a put with the same strike and maturity. Oftentimes the strike is chosen to be as close as possible to the current level of the underlying. The payoff at maturity less the initial costs of the position is shown in Figure 2. From this it is clear that this is a bet on the underlying performing really poorly or very well. Note that it would be impossible to implement such a strategy using the underlying alone or even futures contracts on the underlying.

Looking at the plot it is clear that if the underlying drops, the put option is in the money at maturity and thus pays off. On the other hand, if the underlying increases in value, the call option is in the money, and thus it will pay off. In fact, the worst thing that can happen when being long a straddle is that "nothing" changes. If this happens to be the case both options have zero pay off at maturity and the investor looses an amount equal to the cost of setting up the straddle in the first place.

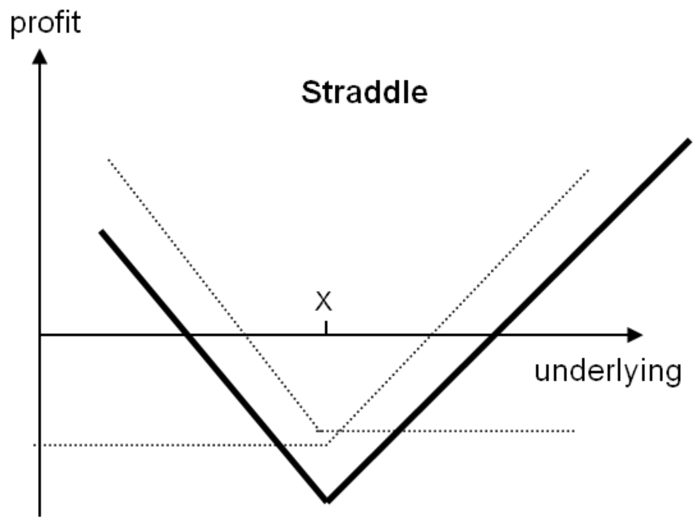



**Figure 2: Profit at maturity, measured as the final payoff minus the cost of setting up the position, of a straddle position as a function of the underlying asset value.**

A straddle position set up with options struck at the current level of the underlying is in general delta neutral or close to delta neutral. That is, there is little directional exposure which may seem odd. To see this remember that the delta of an at the money call option is close to ½ meaning that the delta of the put is ½-1=-½. Thus, this yields an overall delta of roughly zero for a straddle with at the money options.[4]

However, while there is little delta the position is loaded up on gamma and vega. That is, the position is long curvature and volatility. However, this makes sense since large and volatile movements are exactly what will be profitable with this strategy. This also explains why, when setting up the strategy, traders often choose options which are at the money. These, we know, are exactly the ones with the largest values for gamma and vega. Thus, choosing the strike price close to the current value of the underlying maximizes the exposure to volatility.

Note that, as it is always the case with long position in options, the strategy is short theta. That is, the investor is short time. However, we have already seen that if the underlying remains stable throughout the life of the options they will end up with little or no payoff at maturity and there will be no profit. This is exactly what it means being short theta – if nothing else changes but time passing the investor's position which is long volatility "bleeds to death" by theta. As

[4] Strips and straps, on the other hand, are in general either short or long on delta also.

always in the financial markets nothing comes for free. Thus, if the investor wants a lot of vega he or she has to realized that with this comes a lot of theta.

## Bull spreads and the Greeks

A bull spread may be constructed in two different ways. The position may be long a call and short a call with higher strike price, or long a put and short a put with higher strike price. The payoff at maturity less the initial costs of the position is shown in Figure 3. The figure clearly shows that this is a bet that the price of the underlying will increase.

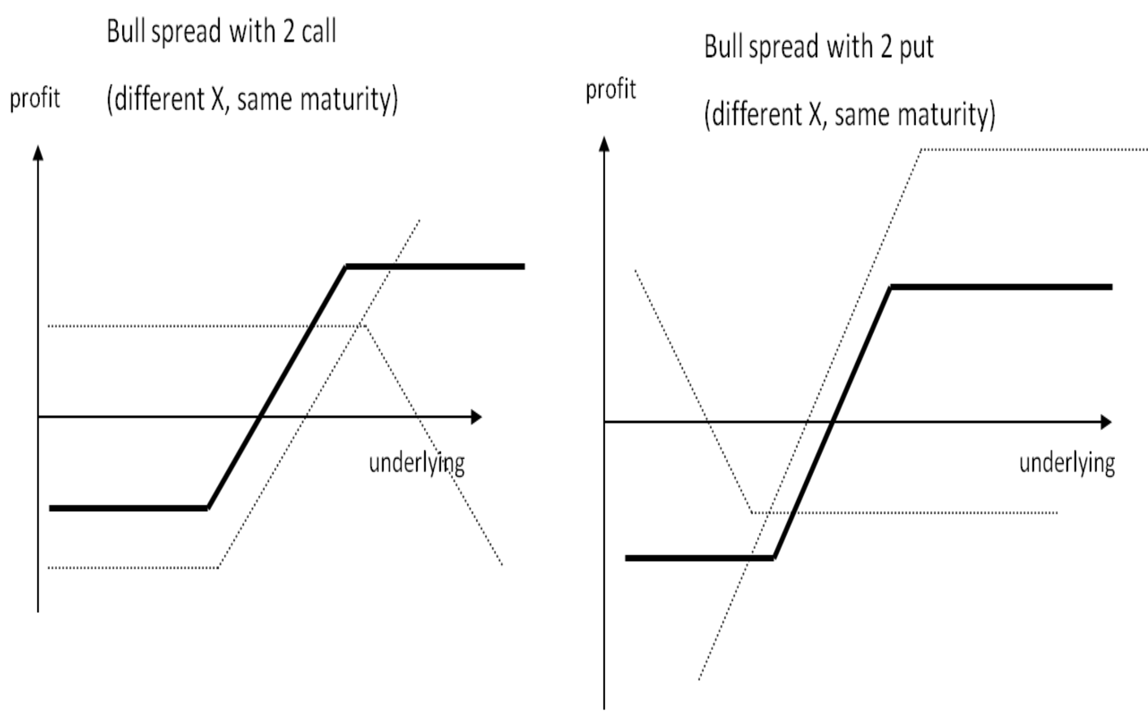



**Figure 3: Profit at maturity, measured as the final payoff minus the cost of setting up the position, of a bull spread position as a function of the underlying asset value. The left hand plot shows a bull spread constructed with 2 call options, whereas the right hand plot shows a bull spread constructed with 2 put options.**

The actual exposure to the Greeks with a bull spread is more complicated than that of a straddle and changes with the underlying. In the following we will focus on the delta and the gamma of a bull spread. The particular example we take is the one constructed with call options. However, since the same payoff profile is obtained with put options this bull spread would have the same exposure to the Greeks also.

Suppose that the underlying is close to the strike price of the long call option. This option is then at the money and has maximum long gamma. The delta is also relatively high and around ½ for this option. For the short call, on the other hand, there is little or no gamma, as this option is far out of the money. Moreover, although this option may have some delta this is in any case much less than that of the at the money long position. Thus, overall the position is long delta and gamma when the underlying is close to $X_1$. When looking at Table 3, which shows the profitable market movements depending on the exposure to the Greeks, we see that for the portfolio to increase in value the underlying should increase in price, long delta, and move quickly, long gamma. This is exactly the reason for implementing the strategy in the first place.

Next suppose that the underlying is close to the strike price of the shorted call option. This option is then at the money and has maximum gamma. The delta is also relatively high and around ½. For the long call, on the other hand, there is little or no gamma left, as this option is far in the money. Moreover, this option has a delta close to one which is in any case much higher than that of the at the money short position. Thus, overall the position remains long delta but is short gamma when the underlying is close to $X_2$. For this to be a profitable portfolio the table in the appendix shows that the underlying should still increase rather than decrease in price, long delta, but it does not need to move quickly, short gamma. Compared to the payoff from the strategy this is exactly as wanted since we are getting closer to the upper bound caused by the shorted option.

## Calendar spreads and the Greeks

Up until now we have consider option strategies which involves options with the same maturity and where the focus is on the movements in the underlying. The calendar spread, as the name might reveal, is related to time and is also sometimes referred to as a time spread. The usual calendar spread involves simultaneously selling options with a given strike price expiring in a nearby month and purchasing options with the same strike price but which expire in a more distant month. The strategy is profitable if the underlying remains unchanged, and it is in this respect similar to the butterfly spread.

When a trader sets up a calendar spread it is often approximately delta neutral and instead the calendar spread is used to obtain exposure to theta, i.e. exposure to time. The reason is that the short term option will have more theta than the long term option, whereas the delta of the two options is roughly the same. The exposure to theta could, of course, also be obtained by just shorting an option. However, in doing so the trader is also very much exposed to delta, i.e. a directional move in the underlying.

It is important to note that with this type of strategy the trader is also exposed to gamma and vega risk. The reason is that not only does the short term option have more theta then the long term option it also has more gamma and much less vega than this option. Thus, the calendar spread in general is short on gamma and long on vega. Therefore, calendar spreads are also used by traders to take advantage of a rise in future volatility as this would increase the value of the long term option more than the short term option. Again by using a calendar spread this exposure is obtained while being delta neutral.

If the trader instead buys the nearby month's options and sells the options which expire in a more distant month with same striking price, this is known as a reverse calendar spread. The reason for setting up this strategy is often related to the overall exposure to vega instead of the theta or gamma exposures. The reverse calendar spread is short vega, meaning that it will be profitable if the volatility of the asset decreases. If this happens the value of the long term option which was sold will decrease much more than the short term option which was bought. Note that short exposure to vega could also be obtained simply by shorting the long term option but this would leave the trader exposed to delta risk as well.

### Diagonal spreads and the Greeks

The diagonal spread is an option strategy that involves simultaneously purchasing and selling an equal number of options of the same type and with the same underlying security, but which have different strike prices and different expiration months. The diagonal spread is in this respect and “extended” calendar spread, and will first and foremost profit from the rapid time decay in options that are close to expiration.

The main difference between the calendar spread and the diagonal spread lies in the near term outlook, and traders who are using diagonal spreads often do this to include some directional exposure in the strategy also since this will not be delta neutral. As an example consider a diagonal spread where the long term option is in the money instead of at the money. Then the overall position is long delta, and thus the strategy will profit from an increase in the price of the underlying also.

## Dynamic option strategies

With futures contracts we have seen that it is possible to perfectly hedge the exposure of the underlying. This is a delta hedge – it is directional. Options can be used for hedging also but this will in general not be “perfect” due to nonlinearities in the price. Specifically, as the price of the underlying changes so does the delta of the option, and hence the overall portfolio cannot then be delta hedged.

For this reason hedging strategies with options are rarely static but instead what is implemented is a dynamic trading strategy. It should be noted that the gamma of the option is a good measure of how dynamic the dynamic strategy has to be, i.e. how often the investor has to rebalance the portfolio. The reason for this is that gamma is the sensitivity of the delta to changes in the underlying. Thus, if gamma is high the strategy will involve a lot of trading through time.

However, dynamic option trading strategies more often occur because traders are hedging positions in options. This is the same as when traders hedge futures positions, but because of the nonlinearities in options this strategy again has to be dynamic. In this section we will consider several strategies which fall within this category.

### Delta neutral gamma long strategy

This is one of the most commonly used trading strategies for option traders. As it is gamma long it consists of a long position in options and this position is then made delta neutral using the underlying. The reason for using the underlying is that this has no gamma and thus does not affect the overall exposure to this particular Greek.

The simplest possible way to implement this type of strategy is longing a call option and then hedging the delta by shorting the underlying. Graphically, the payoff of this position can be illustrated as in Figure 4.

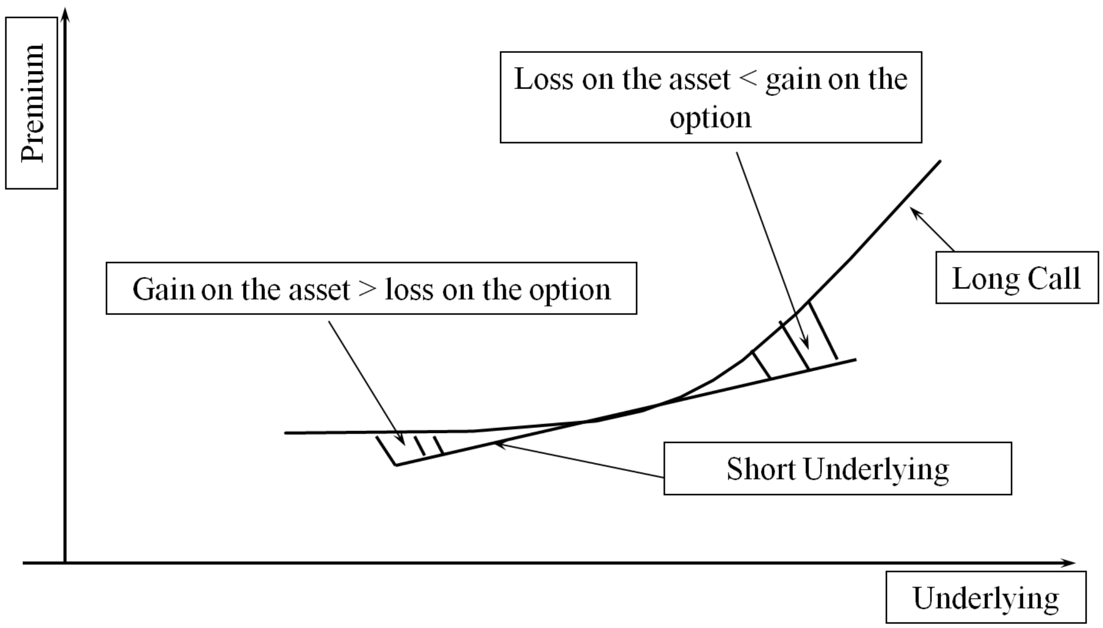



**Figure 4: Example of a delta neutral gamma long position with a long call option which is hedged using the underlying. The figure shows that if the price of the underlying increases the loss on the hedge is less than what is gained on the option position. Likewise, it can be seen that if the stock price decreases the loss on the option position is less than what is gained on the hedge. This is due to the convexity of the option price as a function of the underlying asset.**

From the figure we see that, because of the convexity in the option price curve:

- We make more on the long option position than we lose on the short stock position if the underlying trades up
- Likewise we gain more on the short stock position than we lose on the long option position when the underlying trades down

This profit should be locked in! How should an investor do this? He or she should re-hedge the position so as to leave the portfolio immune to movements in the underlying, that is delta neutral. Dynamically implementing this strategy thus calls for re-hedging the portfolio.

To see what this leads to suppose the underlying has increased in value. This will increased the delta of the option, and thus the dynamic rebalancing requires the investor to sell more of the asset to remain delta neutral. Alternatively, if the underlying has decreased in value dynamically re-hedging requires the investor to buy the underlying. Thus the overall trading amounts to selling stock on the way up and buying it back on the way down.

This sounds a lot like a dynamic trading strategy you may have seen before. In fact the delta neutral gamma long trading strategy is similar to that implied by a Constant-Mix dynamic trading strategy. This strategy is known to be profitable in volatility markets without a trend. However, to make money in such a market one should be delta neutral, since there is no trend, and gamma long since there is volatility. But this is exactly what the above strategy is! All that is required is that the underlying moves through time. If this happens the trader is sure to make money with this strategy.

### *Risks with the strategy*

The risk with the above strategy is that, apart from volatility collapsing and large time decay effects, the trader has not realized that gamma is large. If volatility does increase, and the trader re-hedges using the underlying asset, then he or she certainly won't lose money. However, the point here is that if the trader ignores the high gamma then he or she will not make nearly as much money as could have potentially been made.

## The straddle – another delta neutral gamma long strategy

The above strategy earns money because of gamma and the more gamma the more money can be earned.[5] Now, we have seen that all long options positions have gamma and we thus attempt to delta hedge the long call position with options instead. Specifically, this would involve using put options as these are short delta. Suppose that the call option is bought at the money, since this maximizes the gamma, with a delta of approximately ½. Then the position can be approximately delta hedge with a long position in a put option with the same strike since this will have a delta of -½. But this is exactly the setup needed for a straddle!

Thus, it should come as no surprise that traders, when setting up straddles, will also use a dynamic trading strategy. In particular, they may actively delta hedge the position using the underlying asset. Although, in principle delta hedging is not needed when setup, should the underlying asset move traders will hedge their gain. In other words, option traders will attempt to lock in any profit from moves in the underlying before maturity. In fact, even if the options end up at the money and the straddle expires worthless, with this dynamic strategy money can be made as long as the underlying actually moved, at some time between the time the straddle was setup, and the maturity of the options.

## Some practical issues with delta neutral gamma long strategies

Although, the delta neutral gamma long strategy is a sure money maker this depends critically on the gamma. As an example consider the straddle and assume that the current level of the underlying is much higher than the strike prices of the options. The delta of the option position is now close to one since the calls are deep in the money and the puts are out of the money. This means that the hedging is almost "perfect", and any gains on the option position from movements in the underlying are completely offset by losses on the hedge, and vice versa.

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[5] Although, with more gamma comes more theta!

However, this may not be what the trader hoped for if attempting to exploit the convexity of the options. The problem with this is that there is no gamma left in the options and hence no curvature. Thus, what traders will often to is to close out the straddle and set up a new one using options which have a strike price closer to the present value of the underlying. Doing this, once again, ensures that gamma is maximized.

## Delta neutral gamma short strategies – is this a sure looser?

We have seen above that the delta neutral gamma long strategy is sure to earn money if the underlying asset moves. However, since options are zero sum investments this must mean that the counterparty is sure to lose money. Who is this person who accepts such a bad deal – well often this is the market maker in options. While the market maker collects the premium for the option up front there are risks associated with this job even if one delta hedges the position. It is theoretically possible to eliminate the losses due to being short gamma if one trades often – say continuously. In particular this was the argument we used to derive the actual pricing formula for the option.

An alternative to the continuously trading strategy is to delta hedge and gamma hedge the position. Theoretically, this eliminates the risks associated with exposure to gamma. So how can this be achieved? The first thing to note is that to implement such a strategy it is necessary to gamma hedge first and then delta hedge. The reason is that in order to gamma hedge the trader will need to use options and these will change the delta of the overall portfolio. Thus, the rule is that first you gamma hedge and then you delta hedge.

### A shorted straddle

As an example suppose that a market maker has sold, i.e. shorted, the straddle we have been using as an example above. The market maker stands to earn a profit if the underlying remains stable. This profit is the option premium – which at least theoretically compensates the option writer for the risks associated with the short end of the option deal.

Suppose the market maker wishes to hedge the position which, although approximately delta neutral, is gamma long. This can be done by using any of the other options contracts which are available out there and thus could be achieved in various ways. What should determine the choice? Well, first of all we know that there is most gamma in at the money options. Secondly, we know that the shorter the maturity the more gamma. Both of these factors should thus be considered.

# Summary

Options are interesting from an investment perspective because of their sensitivity to multiple underlying factors and of the price nonlinearities. Because of this exposure to various factors can be obtained using options and interesting strategies can be implemented. In this chapter we discussed various options price sensitivities which are also known as the Greeks. We then

reviewed some of the most important strategies, i.e. straddles and the various spread trades, and analysed these using the sensitivities.

Moreover, because of the nonlinearities oftentimes dynamic trading strategies are warranted when considering options. This holds even in the simplest possible case when using an option to delta hedge a position in the underlying. Moreover, it may be shown that when implementing specific types of strategies dynamically a profit is virtually "guaranteed". This strategy is the delta neutral gamma long strategy.

Finally, it should be noted that options like other derivatives are zero sum type investments. Thus, the gains associated with the delta neutral gamma long strategy is the loss of the holder of a position which is delta neutral but gamma short. We discussed briefly how this position may be hedged. Such a strategy in general involves an additional position in options to hedge the gamma.

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# Appendix: The Greeks from the Black-Scholes formula

Although it is a very simple model, the Black-Scholes model remains the benchmark model used in the industry for option pricing and for the calculation of option price sensitivities. In this appendix we review the formula for the price calculations as well as the formulas needed for calculating the Delta, Gamma, Theta, Vega, and the Rho of European options. The formulas listed here are for options which do not pay dividends, but they can easily be extended to take account of continuous dividend payments. At the end of this appendix we provide some numerical examples of the Greeks.

## The Black-Scholes formula

The Black-Scholes formula for the price of a European call option, C, and put option, P, on a non-dividend paying stock are

$$C = SN(d_1) - Kexp(-rT)N(d_2)$$

and

$$P = Kexp(-rT)N(-d_2) - SN(-d_1),$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, and\ d_2 = d_1 - \sigma\sqrt{T}.$$

The function $N(x)$ is the cumulative distribution function for the standardized normal distribution. In other words it is the probability that a standard normal distributed variable will be less than x. The other variables are the ones affecting the option price. Of these the interest rate, r, the volatility, σ, and time to maturity, T, should be considered at an annualized basis.

## The Greeks

All of the Greeks can be calculated very simply as derivatives of the above formula. That is, to get the Delta we simply differentiate the above Black-Scholes formula with respect to the underlying assets. For the other sensitivities we differentiate with respect to the appropriate variable. In the following we report the actual formulas for each of these sensitivities.

### Delta

Delta of a European call option is calculated as

$$\Delta(call) = N(d_1).$$

Using the same procedure or from the put-call parity the delta of the put is then calculated as

$$\Delta(put) = N(d_1) - 1.$$

In these formulas $d_1$ is as specified above and $N(x)$ is the cumulative distribution function for the standardized normal distribution.

### Gamma

Gamma of an option, be that a put or a call option, is calculated as

$$\Gamma = \frac{\phi(d_1)}{S\sigma\sqrt{T}},$$

where $\phi(x)$ is the probability density function for the standardized normal distribution.

### Theta

Theta of a call option is calculated as

$$\Theta(call) = -\frac{S\phi(d_1)\sigma}{2\sqrt{T}} - rKexp(-rT)N(d_2),$$

and Theta for a put option is calculated as

$$\Theta(put) = -\frac{S\phi(d_1)\sigma}{2\sqrt{T}} + rKexp(-rT)N(d_2).$$

This means that for otherwise equal options Theta is slightly less negative for a put option than for a call option.

### Vega

Vega is again the same for equivalent options and is calculated as

$$Vega = S\sqrt{T}\phi(d_1).$$

### Rho

Rho is given by the following formula for a European call option

$$\rho(call) = KTexp(-rT)N(d_2),$$

and for an equivalent put option it is given by

$$\rho(put) = -KTexp(-rT)N(-d_2).$$

### Numerical examples

The formulas above can easily be implemented in e.g. Excel using built in formulas. The table below show the values for options, a put as well as a call. The numbers used are S=100, K=100, T=½, r=6%, and σ=25%. Using these numbers you should find the following value for $d_1$=0.2581 and $d_2$=0.0813.

| "Greek" | Price | Delta | Gamma | Theta | Vega | Rho |
|---|---|---|---|---|---|---|
| Call Option | 8.516258 | 0.60183 | 0.021828 | -9.9214 | 27.2854 | 25.8335 |
| Put Option | 5.560811 | -0.39817 | 0.021828 | -4.0987 | 27.2854 | -22.6888 |