Chapter 9: A Brief Introduction to Derivatives

Reading this chapter you will be introduces to the following key concepts:

- The most important types of derivatives: plain vanilla futures and options
- Valuation of derivatives: arbitrage pricing, Black-Scholes-Merton and binomial models
- Simple static trading strategies using futures: arbitrage, hedging and speculation
- Simple static trading strategies using options: how to trade time and volatility, combinations and spreads

Introduction

Option and futures contracts both fall within the general category of derivative instruments. This is a very large group of investment instruments which these days has almost a bad name. The bankruptcies of Orange County and Barings Bank, the cases of Long Term Capital Management, and the current credit crises all involve derivatives, and oftentimes these instruments are being blamed for the severity of the crashes or crises. However, just as the saying “guns don’t kill people, people kill people” one might postulate that “derivatives don’t cause crises, people cause crises”.

The reason that options and futures are referred to as derivatives is that they derive their value from other securities, which are referred to as the underlying security. Thus, whereas shares entitle the holder to future dividends, as such derivatives have no economic value. Moreover, investing in derivatives is a “zero-sum” game in the sense that if you are long a derivative somebody else is short that same contract and summing all the positions gives a big zero! So you may wish to know what purpose derivatives serve. The answer is that they fundamentally serve to transfer market risk from one participant to another.

Derivative contracts are also fundamentally different from e.g. shares in the sense that there is no physical ownership attached to the contracts. Thus, the payment on e.g. a long option position comes from the holder of the short position. That is, to each position there is a counterparty, and thus there is potentially a risk that this counterparty will default. If this happens the contract becomes worthless! In order to avoid this, the derivative markets are heavily regulated.

The size of the world market for derivatives has been estimated at about $791 trillion face or nominal value, roughly 11 times the size of the entire world economy. This should be compared to the stock market which was estimated at about $36.6 trillion US at the beginning of October

2008. Note though that the value of the derivatives market, because it is stated in terms of notional values, cannot be directly compared to a stock or a fixed income security, which traditionally refers to an actual value. However, it does show that this market is extremely important.

In this document we will focus on the simplest possible types of derivatives, futures and options, and we will use the generic example of the case of derivates on a stock market index for illustrative purposes. We will start by defining the concepts, and we then discuss their valuation. The major part of this document is concerned with discussing the use of derivatives for trading and the potential trading strategies, which may be put in place using this class of instruments.

Definitions

Of the two plain vanilla type derivatives contracts, futures and options, the futures contract is the simplest one.[1] We will use the following definition for such a contract:

> **Definition:** A futures contract refers to a standardized contract to buy or sell a specified asset (the underlying asset), at a certain date in the future (the maturity date), at a predetermined price (the futures price).

Thus, if you enter into a long futures contract on the S&P 500 index which matures in May at a price of 1200; this means that come May you are obligated to buy the S&P 500 index at 1200 from the seller of the futures contract. For the time being, we will abstract from the fact that you actually cannot buy the S&P 500 and that financial futures, such as those on stock indices, are cash settled and that no delivery takes place of the actual asset. However, these are important elements of the actual contract specification. Note also that no money changes hand when the contract is entered into as all cash flows happen at the maturity date.

The two types of European style options, put and call options, may be defined as:

> **Definition:** A call option grants the buyer the right but not the obligation to buy a particular asset (the underlying asset), at a certain date in the future (the maturity date), at a predetermined price (the strike price).

> **Definition:** A put option grants the buyer the right but not the obligation to sell a particular asset (the underlying asset), at a certain date in the future (the maturity date), at a predetermined price (the strike price).

Thus, if you buy a call option on the S&P 500 index which matures in May and has a strike price of 1200; this means that come May you have the right to buy the S&P 500 index at 1200 from the seller of the futures contract. Thus, it is immediately seen that the major difference between the futures contract and the option contract is that the first is an obligation to trade whereas

[1] The term “plain vanilla” simply means that these assets have no special features.

the latter is a right or an option! Because the option contract is a right and not an obligation it has a nonzero positive value (the option price or option premium).

The corresponding American style options may be exercised, i.e. the underlying may be bought or sold, at any time prior to the maturity date in addition to at the actual maturity date. Thus, these options give the option buyer more options and hence are generally more valuable. In fact such options are rarely exercised and instead they are sold back to the issuer. The reason for this is that it is easier to do this cash transaction, than it is to physically deliver or take position of the underlying asset and then to transfer the ownership to the counterparty.

Valuation

The underlying principle behind the valuation of derivates is the **absence of arbitrage** or the **law of one price**. In plain English this means that, if I can purchase a good today for a price *S* and conclude a contract to sell it one month from today for price *F*, the difference in price should be no greater than the cost of using money minus any expenses (or earnings) from holding the asset. If the difference is greater, I would have an opportunity to trade the "spot asset" and the "futures" for a risk-free profit. In highly liquid and developed markets, we would not expect this potential for arbitrage to exist for long, and hence this theoretical relationship should hold.

Futures contracts

The principle of no arbitrage pricing is best illustrated with the example of the futures contract defined above. Consider an investor with a long position in the underlying and assume that this is currently trading at *S* and a short position in the futures contract at a price of *F* at time t. Because of the contract specification the overall value of this portfolio at maturity equals *F* irrespective of the value of the underlying at maturity. Since this is known at time t the actualized cost of the portfolio should equal this value. The cost of setting up the portfolio is simply *S* and thus if the contract matures in T periods, at time t+T, and the interest rate until this time is r it must be the case that

$$F_t = S_t \exp(rT).$$

In this relationship *exp(rT)* is oftentimes referred to as the **cost of carry**, which refers to the cost of having to pay cash for the asset up front at time t compared to buying it according to the futures contract at the later time T. This cost corresponds to the **opportunity cost** of not earning interest on this amount.

The above formula is referred to as the **spot-futures parity**. The spot-future parity condition does *not* say that prices *must* be equal (once adjusted). Rather it states that when the condition is not met, it should be possible to sell the expensive asset and purchase the other asset which is relatively cheap for a risk-free profit. That is, to undertake arbitrage. As an example, suppose that the parity does not hold and that $F_t < S_t \exp(rT)$. An arbitrageur should then buy the futures contract and sell the underlying asset (perhaps by shorting it using his margin account). This would give him S_t at time t. Depositing this money in a bank account at an interest rate of r it will grow to $S_t\exp(rT)$ at time T. At this time however, the arbitrageur is required to buy the underlying at a cost of F_t and he or she can return it to the broker. When the transactions are over the arbitrageur is left with $S_t\exp(rT)-F_t>0$ dollars of risk free profit.

General versions of the spot-futures parity

The spot-futures parity developed above was derived for the simple case of a financial asset paying no dividends. However, the relationship may be generalized easily to other more general cases. One such example is for a stock which pays a continuous dividend q. In this situation the relationship would be

$$F_t = S_t \exp\big((r-q)T\big).$$

Again, in this relationship exp((r-q)T) is the cost of carry. Since dividends are paid when holding the asset and not when holding the futures contract it appears as a negative cost, i.e. a yield rather than a cost.

In its most general and complete form the spot-futures parity is given by

$$F_t = S_t \exp\big((r+y-q-u)T\big),$$

where r is the applicable interest rate, y is the storage cost over the life of the contract, q are any dividends accruing to the asset over the period between the spot contract (i.e. today) and the delivery date for the futures contract in yield, and u is the convenience yield, which includes any costs incurred (or lost benefits) due to not having physical possession of the asset during the contract period. In general some but not all of these costs or yields may be important.

Limitations to the use of arbitrage pricing for futures contracts

Strictly speaking, the spot-futures parity only holds for *investment* assets. That is, assets which are held for investment purposes by a significant amount of investors. Stocks and bonds are clearly investment assets, but so are gold and silver. Thus, investment assets do not have to be held exclusively for investment, but there has to be a significant amount of investors who use it for investment purposes so that arbitrage can be rule out.

For *consumption* assets the relationship above may not hold with equality. Instead it may only provide an upper limit to the futures price. The reason is that the commodities have value as they may be used, i.e. consumed. This is not the case with the futures and hence such a contract commands a lower price. Examples of consumption assets are most of the commodities, such as copper and platinum, and agricultural products, such as wheat and pork bellies.

The case of oil – purely consumption asset or also an investment asset:

Crude oil is the world's most actively traded commodity. Since it was "discovered" in 1859 the world at large has become extremely dependant on this "black gold" and countless wars have been fought over it. In particularly the two wars against Iraq in 1991 and 2003 had close connections to oil. Moreover, powerful organisations such as the Organization of the Petroleum

Exporting Countries (OPEC) exist, and when they cut production along with imposing embargo for various countries this lead to the Oil Crisis in 1973.

Oil is probably not interesting from an investment point of view because of the huge storage costs, among other things. However, the importance of oil should not be neglected, since for many industries, even countries, the price of oil influences future performance.[2] As this price fluctuates, companies face risks. The various derivatives contracts traded on exchanges around the world serve an important purpose for managing this risk. These contracts may be used by government entities, corporations, and even individual investors. Their extensive use explains why this consumption asset has investment characteristics and hence the spot-futures parity may apply with little error.

Option contracts

Because of the special feature of the option contract it is generally not possible to construct a portfolio containing only a single option and the underlying which yields a certain future payoff, although we will see one example of this shortly. Instead, in order to do this both a put option and a call option is needed. Thus, consider a portfolio consisting of a long position in the underlying, a long position in a put option with a strike price equal to K, and a short position in a call option with a strike price equal to the same K. The future value of this position is equal to K irrespective of the value of the underlying. This is easy to realize when looking at the combined payoff at maturity which is given by

$$S + \max(K - S, 0) - \max(S - K, 0) = K.$$

Thus, with the absence of arbitrage the present value of *K* in T periods, at time t+T, should equal the cost of setting up the portfolio. That is

$$S_t + P_t - C_t = K * exp(-rT).$$

This relationship is referred to as the **put-call parity**. It holds for European style options only though. For American style options only bounds on the prices can be obtained, the reason being that it may become optimal to exercise the options before maturity.

Sometimes the Put-Call parity is re-written as

$$S_t + P_t = K * exp(-rT) + C_t.$$

This relationship states that the following two portfolios should have the same value:

1. One European put option together with the asset
2. One European call option together with Kexp(-rT) in cash

[2] As an example consider the numerous airlines which used the derivatives markets to hedge future expenditures when oil hit record high values during the recent crises.

However, this makes complete sense since at maturity both of these portfolios will be worth max(S_T,K). Thus, in the absence of arbitrage they should cost the same today. This is, of course, just another way of proving the put-call parity.

Besides the put-call parity it is difficult to be specific about option prices without adding more structure and thereby making potentially restrictive assumptions. However, we may quantify the six factors affecting the price of stock options. These are:

1. The current stock price, S_0
2. The strike price, K
3. The time to expiration, T
4. The volatility of the underlying asset, σ
5. The risk-free interest rate, r
6. The dividends expected during the life of the option, q

Table 1 illustrates how the option price changes when either of these factors changes holding the other factors constant.

Table 1: Summary of the effect on the price of a stock option of increasing one variable while keeping all other fixed. In the table a "+" means that when this factor increase the relevant option price increase, a "-" means that the option price decrease, a "?" means that the effect on the option price is uncertain and may go in either direction.[3]

Variable	European call	European put	American call	American put
S_0	+	-	+	-
K	-	+	-	+
T	?	?	+	+
σ	+	+	+	+
r	+	-	+	-
q	-	+	-	+

The table shows that in general option prices are affected in the same direction whether these are American style or European style. The table also shows that in general put and call options are affected inversely by the factors with the exception being for changes in volatility. Volatility is special because it increases the spread of future asset prices. However, since options are "asymmetric" in their payoff, only the increased "upside" potential is important. Hence prices for put as well as call options increase when volatility increase.

We may go a little bit further though in terms of quantifying the actual price of an option. Let us consider a call option before it matures. The value of exercising the option, known as the **intrinsic value**, is max(S-X,0). However, even if S<X and the intrinsic value is zero we often

[3] For a detailed explanation and intuition about the effect of all these factors see e.g. *Options, Futures, and other Derivatives* by John C. Hull.

observe that the option trades at a positive value. The reason for this is **time value**. This value stems from the fact that even for deep out of the money options there is a positive probability that the price of the underlying may rise above the exercise price before the expiration of the option. This would bring it in the money and thus lead to a positive payoff at maturity. Note that the longer the maturity the more significant is the time value. It is the time value which is difficult to estimate, and in general in order to do this one will have to make specific assumptions about market conditions and potentially also about investor behaviour. The relationship is illustrated in Figure 1 below.

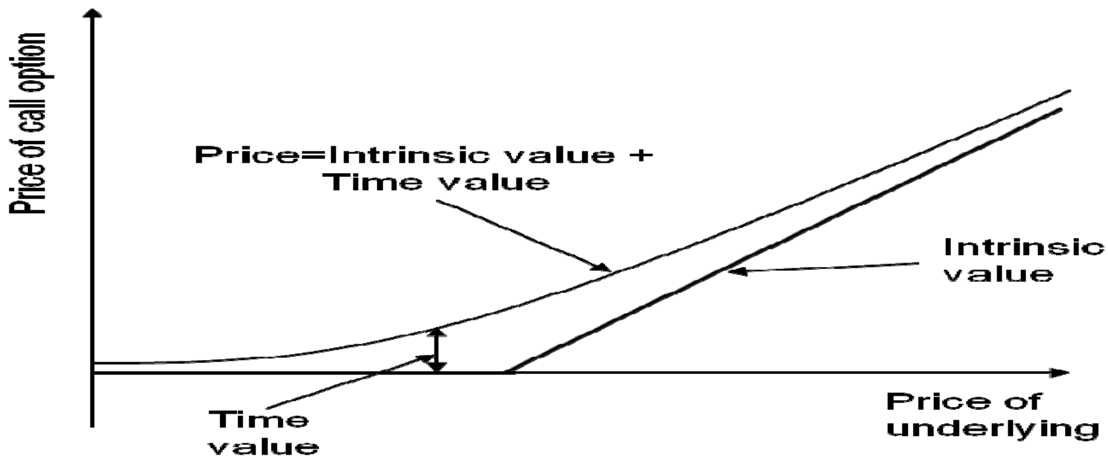

Figure 1: The price of a call option as a function of the value of the underlying. The figure shows that the price is the sum of the intrinsic value, the value of exercising the option immediately, and the time value, the value associated with the possibility, however remote, that the option may increase in value due to volatility in the underlying asset.

Two special situations – the binomial model and the Black-Scholes-Merton model:

There are two special cases under which it is possible to derive "simple" ways to price options. The first of these is in the special case where trading only happens at discrete times and between these times stock prices can move to only two future states. This is the binomial model.[4] The second case is when the stock returns are assumed to be Gaussian and trading is possible in continuous time, i.e. at any given point in time. This leads to the seminal Black-Scholes-Merton model.[5]

The binomial model

Suppose that trading happens at discrete points in time, say at the end of each trading day, and that the price of the underlying at each point either can move up or down. Let these movements be denoted by u and d, such that if the current stock price is S and it moves up tomorrow it will end at uS. Alternatively it can move down to dS. This is arguably the simplest possible way to

[4] The binomial model was first proposed by Cox, Ross and Rubinstein in 1979.

[5] This model was proposed simultaneously by Fischer Black and Myron Scholes and by Robert Merton in 1973. For this Merton and Scholes received the Nobel Prize in Economics in 1999 (Fisher Black had passed away in 1995).

model uncertainty! In addition to the stock assume that investors have access to a savings account earning them an interest of r.

Next, suppose that there exist an option with value f now and value f_u if the stock moves up and f_d if it moves down, respectively. Now, if an investor forms a portfolio by shorting the option and buying h units of the stock then this has a value of hSu-f_u if the stock moves up and hSd-f_d if the stock moves down. Equating these two it turns out that the portfolio is riskless if

$$hSu - f_u = hSd - f_d \iff h = \frac{f_u - f_d}{Su - Sd}.$$

Now, since the portfolio is riskless the present value of future payoffs should equal the cost of setting up the portfolio meaning that, as an example,

$$hS - f = (hSu - f_u)\exp(-rT).$$

Substituting the expression for h above into this and simplifying we obtain the following expression for f

$$f = \exp(-rT)\left[\frac{e^{rT} - d}{u - d} f_u + \frac{u - e^{rT}}{u - d} f_d\right].$$

It is seen that the option price depends only on S, u, d, and r. Although S and r are known, values are needed for u and d. However, it turns out that these are linked to the volatility, the risk, of the underlying asset. The approach used in Cox, Ross and Rubinstein is to set $u = \exp(\sigma\sqrt{t})$ and $d = 1/u$ which ensures the correct level of the volatility.

Thus, it is seen that the binomial model can be used to price options in general as we have not specified the f function. That is, if the option is a call option $f = \max(S - K, 0)$, and hence as an example $f_u = \max(Su - K, 0)$. Note also that the binomial model can easily be generalized to multiple steps. Specifically, by the choice of u and d it follows that the stock price after an up move followed by a down move is the same as that after a down move followed by an up move. Thus the "knots" recombine, and at each knot an iterative version of the formula above can be used to calculate the option value recursively. The generalized version of this formula is simply given by

$$f_t = \exp(-rT)\left[\frac{e^{rT} - d}{u - d} f_{u,t+1} + \frac{u - e^{rT}}{u - d} f_{d,t+1}\right].$$

Here the generalized version of the payoff function should be calculated from $f_{u,t} = \max\left(S - K, f_{u,t}\right)$, if one is pricing an American style option, since this can be exercised early. For the European style options the payoff function, e.g. $f = \max(S - K, 0)$ for a call option, is only used at the very last step. In all other steps the value used is simple given by $f_{u,t} = f_{u,t}$.

The Black-Scholes-Merton model

Whereas the binomial model is based on simple but tedious mathematic calculations, the Black-Scholes-Merton, or BSM, model is based on complex but elegant mathematical derivations. The two most important underlying assumptions of the model is that the returns of the underlying are Gaussian distributed with constant mean and constant volatility and that trading happens in continuous time. Under these assumptions it can be shown that a riskless portfolio can be derived. Because it is riskless it should earn the risk free interest rate as return. Although the math is more complex it uses the same assumptions to derive the following formula for a European call option

$$c = S \cdot N(d_1) - Kexp(-rT) \cdot N(d_2),$$

where

$$d_1 = \frac{\ln(S/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, and\ d_2 = d_1 - \sigma\sqrt{T},$$

and $N(\cdot)$ is the standard Normal Cumulative Distribution Function. For the put option the formula is given by

$$p = Kexp(-rT) \cdot N(-d_2) - S \cdot N(-d_1).$$

Under certain assumptions it is possible to show that the two methods, the binomial model and the BSM, yield the same results. In this case, the BSM method, as it provides an explicit formula, is much faster. However, the binomial model can be adapted to price American style options also as shown above. This though is generally impossible with the BSM model.[6]

Trading strategies with derivatives

In the following we discuss a number of different trading strategies involving positions in the two plain vanilla type derivatives contracts discussed above. For the time being we do not consider trading strategies involving combinations which involve actively trading the underlying and the derivatives in a dynamic way.[7]

Trading strategies involving futures contracts

When it comes to trading futures it is important to realize that this is, in many respects, exactly the same as trading the underlying. This should be obvious when considering the spot-futures parity which provides a theoretical link between the prices of each of these assets. Specifically, going long a futures contract yields the same payoff as going long the underlying at maturity.

[6] It can be shown that for call options on stocks which pay no dividend it is never optimal to exercise these early. Thus, in this case the BSM model may be used.

[7] These strategies are considered advanced and are discussed in Chapter 10: Advanced Option Trading Strategies.

However, the futures market is an important alternative to the spot market for at least three reasons. First of all, sometimes it is easier to trade in futures. For example, you cannot long the S&P 500 index! Secondly, an extra feature of the futures market is that these are generally very liquid and the contracts are oftentimes standardized. And finally, it may be less costly to invest in futures contracts than in the corresponding spot asset.

Among the potential goals of futures trading are the following:

1. Exploit arbitrage possibilities – by using the spot-future parity relation.
2. Hedging – by using futures contracts to protect against future price movements.
3. Speculating – by using futures contracts to profit from future price movements.

It is important to note that hedging and speculating are two polar uses of futures markets whereas arbitrage exploits market inefficiencies only!

Arbitrage:

It is possible to exploit arbitrage possibilities given by the spot-futures relationship. To be specific, consider an investment asset for which the theoretical spot-futures parity should hold with equality. An arbitrageur should exploit the potential arbitrage possibilities in the particular way it was explained above. However, arbitrage strategies can be implemented in other situations also.

First of all, different markets can be arbitraged as futures contracts may exist with the same underlying but traded at different exchanges. A classical example of this is contracts on the Japanese Nikkei 225 index which are traded both in Osaka and in Singapore. Nick Leeson, arguable one of most notorious names in finance, was supposed to exploit potential arbitrage between these two markets for Barings Bank.[8]

In addition, arbitrage may involve futures contracts on the same underlying but with different maturities. This is something which may be implemented in e.g. the fixed income market. And finally, for some commodities derivatives exist on a number of end products derived from the same basic commodity. This allows an additional type of arbitrage which involves operations with these different end product materials. The goal of this type of arbitrage is to obtain profit from a change in the futures price of the materials and the futures price of the final product. Classical examples are for contracts on oil and heating oil and for contracts on soya beans and soya bean oil. The specific spread trade which involves simultaneously buying and selling contracts in crude oil and one or more derivative products, typically gasoline and heating oil is often referred to as the “crack spread”.[9]

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[8] In fact instead he had an overall long position in the market. Thus, when the Nikkei 225 fell following the Kobe earthquake his position lost massively. This eventually led to the bankruptcy of the bank!

[9] The reason for this is that it is related to the profit margin that an oil refinery can expect to make by "cracking" crude oil, i.e. breaking its long-chain hydrocarbons into useful shorter-chain petroleum products.

Hedging:

Because investing in futures yields the same payoff as investing in the underlying, futures can be used to hedge a position in the underlying by taking the offsetting position in the derivative. Thus, if an investor wishes to hedge an already existing position this is easily done by taking the opposite position in the futures market. As an example, suppose that the investor has constructed a portfolio which follows the S&P 500 index but is currently worried that the S&P 500 will fall. As an alternative to selling the entire portfolio and incurring massive trading costs he or she could short a futures contract on the S&P 500 instead. This is much cheaper as only the margin needs to be paid. If the S&P 500 falls in value so will the short position in the futures contract and overall the investor is hedged against this. Off course the upside potential is equally limited since any gains on the portfolio are offset by losses on the hedge portfolio.

Perfect hedging?

In the hedging literature a **perfect hedge** is one that completely eliminates the risk. Unfortunately such hedges are rare. In the case of the stock market index the short position is a perfect hedge only when the horizon of the investment corresponds to the maturity of the futures contract. Another problem in terms of obtaining a perfect hedge occurs when the asset whose price is to be hedged may not correspond exactly to the underlying asset of existing and liquidly traded futures contracts.

Thus, the first problem when hedging is that sometimes, actually more often than not, the horizon over which a hedge is wanted is later than the expiration of all the existing futures contracts. When this is the case the hedger must roll the hedge forward by closing out one futures contract and taking the same position in a futures contract with a later delivery data. Such hedges are rarely perfect.

Secondly, the asset which should be hedged may not have corresponding futures contracts. This is termed **cross hedging**, which refers to the situation when the underlying asset and the asset to be hedged are not the same. The example used in footnote 2 of airlines hedge the risk in future jet fuel prices is in fact an example of this. The reason is that there are no futures traded on jet fuel and the airline company may thus be required to use e.g. futures on heating oil for hedging purposes.

When a different asset is used the hedger will need a **hedge ratio**, the ratio of the size of the position taken in futures contracts to the size of the position to be hedged. It can be shown that the hedge ratio which minimizes the variance depends on the correlation between the assets as well as their variance. It is given by

$$h = \rho \frac{\sigma_S}{\sigma_F},$$

where ρ is the correlation, σ_S is the standard deviation of the changes in the asset to be hedged and σ_F is the standard deviation of the changes in the underlying asset. Note that if the two assets are the same the optimal hedge ratio equals 1!

Speculating:

Because investing in futures yields the same payoff as investing in the underlying, futures can be used for speculation. Thus, if an investor wishes to obtain exposure to the overall market in the US, say, this is easily obtained by going long a futures contract on the S&P 500 index. If the investor is correct and the value of the S&P 500 increase so does the value of the futures contract. In particular this follows from the spot-futures parity. Thus the investment will be profitable. However, note that the actual change in price of the futures contract may not be one to one compared to the change in the spot price due to the multiplicator in the parity. This is, of course, only a minor detail.

An added benefit of using the futures contract is that this may in fact be cheaper than taking a direct position in the spot market even if the underlying is traded. The reason is that the immediate investment in a futures contract is limited to the margin that needs to be paid to the exchange or broker. If this is 20% this means that the investor using the same amount of money can by 5 times the futures contract than the actual underlying. Thus, it is possible to leverage the investment because only the margin needs to be paid. Of course, this investment is much more risky also. In fact, such a position corresponds to buying the underlying on the margin and using leverage!

Finally, using futures it may in fact be possible to speculate directly in assets which are in general relatively illiquid or which are not traded at all. An example of the first case is government bonds which are for the majority traded over the counter, or OTC, and this only in large denominations. This may make these assets relatively inaccessible for the average investor. Luckily futures contracts exist on a variety of such assets and these are readily traded on exchanges. Thus, for the individual investor the existence of futures contracts increases the available investment universe. An example of the latter case is weather derivatives, i.e. contracts where the underlying is relate to the weather and could be the amount of rain, snow or the average temperature. This underlying asset is not traded and the futures contracts allow e.g. farmers to hedge against a poor harvest caused by drought or frost.

Trading strategies with options

While trading futures is like trading the underlying, it turns out that trading options is quite different. First of all, as we have seen previously the price of an option is influenced by a number of factors in addition to the current level of the underlying. Thus, using options it is in fact possible to “trade” these factors. The next section discusses these issues in more detail. Secondly, because multiple options exist on the same underlying but with different exercise prices when advanced payout profiles can be obtained using multiple options. The second section discusses this in detail.

Trading strategies using a single option

As we saw in Table 1 an option’s price is influenced by six different factors, and the table summarize the effect of each of these factors on the option’s price. Thus, when holding an option the investor is exposed to each and every one of these factors. However, from a trading perspective the fact that an option's value is affected by these factors means that by using options each of these factors can be traded. For simplicity here, we will only assume that the interest rate and the dividend remain constant through time. Because of this neither of these is considered for trading purpose. The same holds for obvious reasons for the strike price which is also constant.

Trading the underlying using options

The first of the factors, the price of the underlying, can be "trade" using options just as it was the case for the futures contract. That is, if an investor expects that the price increase a long position in call options or short position in put options should be taken. However, when using options it is important to keep in mind that the relationship between the stock price and the option is highly nonlinear. The exception is when the option is deep in the money in which case the value is approximately linear as Figure 1 shows.

As an example consider a call option and let us assume that the investor expects the underlying asset will increase. The investor has a choice about which option to use, i.e. what should be the strike price of the option. If the investor is correct and the price increases, the corresponding change in the option's value depends on the moneyness of the option as Figure 1 clearly shows. If the option is deep in the money the change is almost one to one whereas for options deep out of the money the option's value changes much less. So should the investor go with the deep in the money option? Well in terms of the return on the investment it is quite possible that because the out of the money option is much cheaper it may in fact offer a competitive return when compared to the much more expensive in the money option.

Trading time using options

In Table 1 it is shown that time to maturity also affects the option price. So does that mean we can trade time? This sounds weird – agreed since the maturity decreases deterministically for an option. However it is possible in a certain sense if everything else remains constant. Intuitively this is inversely related to the time value which was shown in Figure 1.

To be somewhat more specific, let us consider an investor who believes that the only thing which is going to happen until maturity of the option is that time is going to pass. That is, the investor believes in particular that the underlying is going to remain constant and so are all the other factors in Table 1. How can options be used to make money in this setting? Well, let us assume that the investor sells out of the money options. Is this a profitable strategy? Yes indeed. The reason is that at maturity the options expire out of the money and no payout needs to be made from the investor whereas the time value was earned when selling the options initially. That is, it is possible to trade time!

Trading volatility using options

Finally, of all the factors volatility is probably the most interesting. In particular, this factor is unique to the options and does not affect e.g. futures contracts. So the question is how options allow us to trade volatility and what does this mean. It is in fact almost as weird as speculating in time right? Well, it turns out that the way to think about trading volatility is the same as when trading time. It is an “everything else equal” kind of argument which intuitively is related to the time value of an option.

To be more specific, let us consider an investor who believes that the only thing which will change between today and tomorrow is that the asset will become more volatile. That is σ is going to increase. How can options be used to make money in this setting? Well, let us assume that the investor buys options, any options in fact. Is this a profitable strategy? Yes indeed if volatility increases sufficiently. The reason is that for a sufficiently large increase in the volatility tomorrow the options are going to be more valuable even if the stock price did not change at all. That is, it is possible to trade volatility!

Trading strategies using multiple options

In the following we will discuss a number of trading strategies which involves positions in multiple options alone. That is, no position is taken in the underlying. We will examine the payoff from such strategies as a function of the underlying and discuss when such strategies may be beneficial. This last point is obviously related to the investor’s anticipation of future market behaviour.

Combinations:

Combinations are strategies which require simultaneous taking positions in call and put options on the same underlying. The most important strategies we will discuss are the following:

1. Straddles
2. Strips and Straps
3. Strangles

Straddles:

A straddle is a position which is long a call and long a put with the same strike and maturity. Oftentimes the strike is chosen to be as close as possible to the current level of the underlying. The payoff at maturity less the initial costs of the position is shown in Figure 2. From this it is clear that this is a bet on the underlying performing really poor or very well. Note that it would be impossible to implement such a strategy using the underlying alone or even futures contracts on the underlying. In particular, if the underlying drops, the put option is in the money at maturity and thus pays off. On the other hand, if the underlying increases in value, the call option is in the money, and thus it will pay off. In fact, the worst thing that can happen when implementing a straddle is that “nothing” happens and the underlying remains at the initial value. In this case both options have zero pay off at maturity and the investor looses an amount equal to the cost of setting up the straddle in the first place.

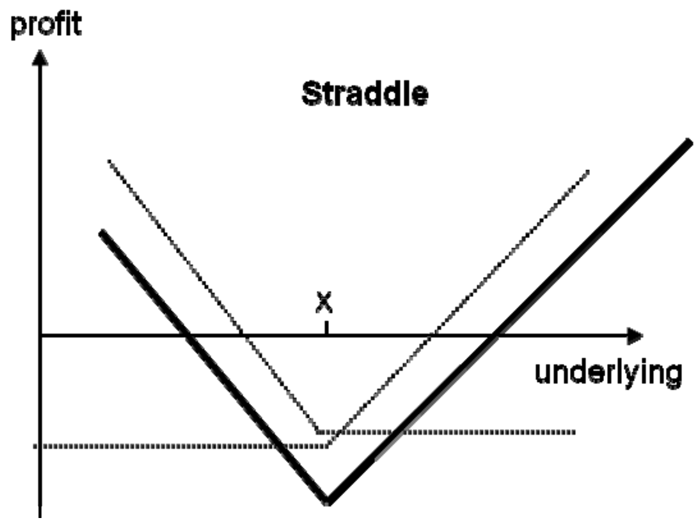

Figure 2: Profit at maturity, measured as the final payoff minus the cost of setting up the position, of a straddle position as a function of the underlying asset value.

Short positions:

The positions above are a long position in options as both the put and the call option are bought. It is obviously possible to short a straddle. This then, would be a bet on nothing happening since any movement would be costly. Specifically, as is always the case with short positions in options these are very risky. This holds doubly with the short straddle as there is "infinite" downside risk on both sides.

As an example of this just remember Nick Leeson, who took short straddle positions when chasing losses he had run up for his employer, Barings Bank. He had initially invested in futures on the Nikkei 225 stock index although his mandate in fact was to do arbitrage between the exchanges in Osaka and in Singapore only. Following the dramatic fall in the market, largely due to the Kobe earthquake, Leeson lost millions as he, instead of being market neutral, was long an incredible amount of contracts. To cover his margin calls he decided to sell straddle positions as shorting options is the only way to obtain money up front in the derivatives markets. He bet that the Nikkei would stabilise and stay in a range around 19,000 points and this was the strike of his straddle. His bet failed and the losses escalated. Eventually this caused the bankruptcy of Barings Bank.

Strips and Straps:

Whereas the straddle uses one put and one call the strips and straps use two or more puts, respectively calls. This increases the potential profitability of movements in one of the directions but in doing so the strategy is more costly to implement. These strategies would be interesting to implement for an investor who has a "biased" or informed view about the future movements. As opposed to this the straddles are used when the investor has no idea at all about the actual direction of the price change.

Strangle:
A strangle is a position which is long a call and a put but with different strike price. Specifically, the call option has a strike price which is higher than the put option and generally also higher than the present value of the underlying asset. Likewise the strike price of the put is chosen lower than the current level of the underlying asset. Figure 3 shows the payoff from this strategy at maturity. From this it is clear that this is the same type of bet which is profitable only if the underlying moves a lot. In fact it has to move by a much larger amount than with the straddle. However, this strategy also costs less to implement as both options are out of the money at the time the strategy is implemented. Thus, in terms of the actual returns this may be an interesting alternative to the straddle.

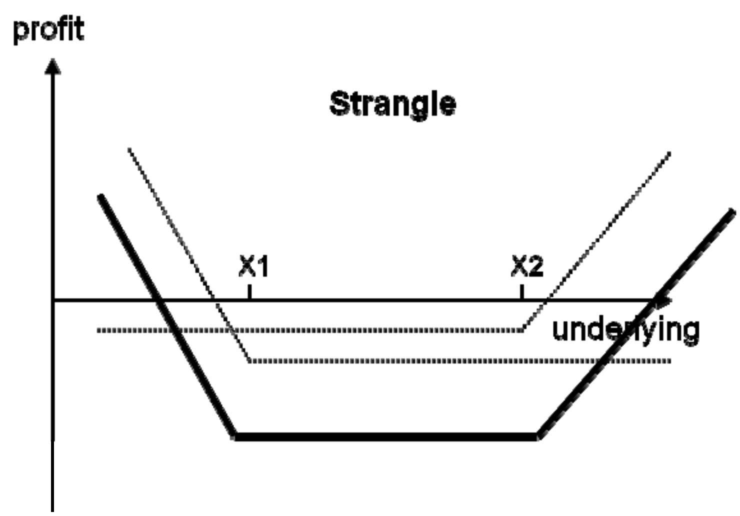

Figure 3: Profit at maturity, measured as the final payoff minus the cost of setting up the position, of a strangle position as a function of the underlying asset value.

Spreads:
Spreads are strategies which require taking a position with 2 or more options of the same type, i.e. calls or puts, but not in both types. The most important strategies we will discuss are the following:

1. Bull spreads
2. Bear spreads
3. Butterfly spreads

Bull spreads:
A bull spread may be constructed in two different ways. The position may be long a call and short a call with higher strike price, or long a put and short a put with higher strike price. The payoff at maturity less the initial costs of the position is shown in Figure 4. The figure clearly shows that this is a bet that the price of the underlying will increase.

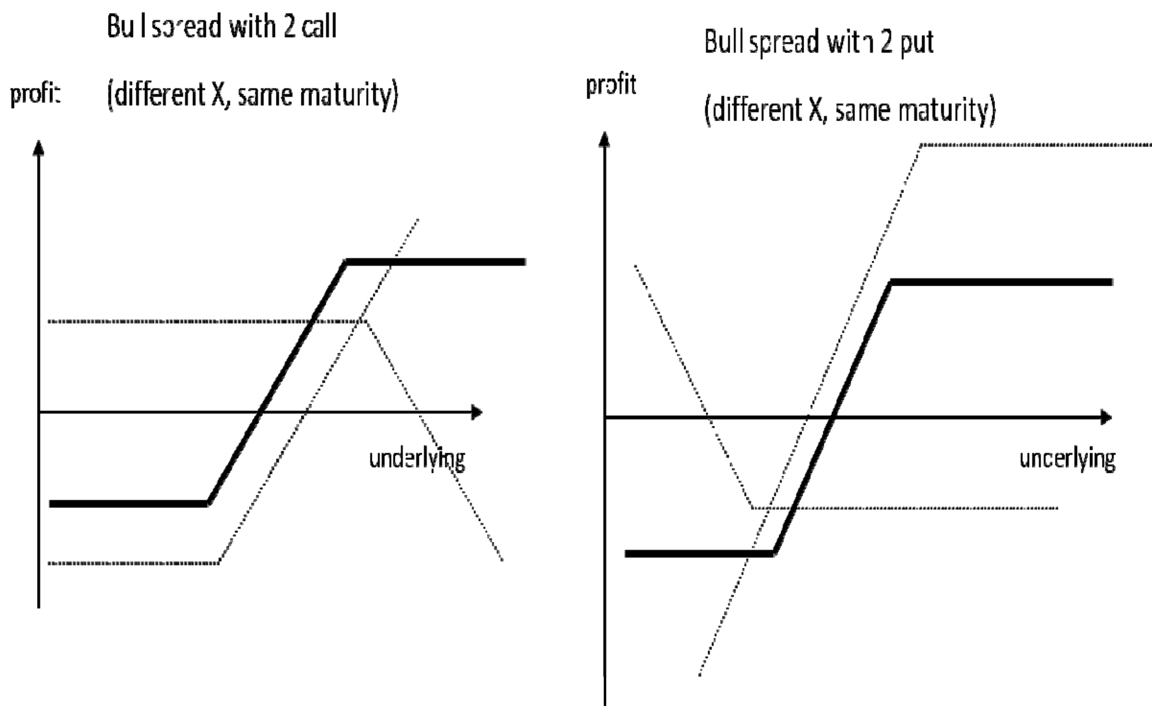

Figure 4: Profit at maturity, measured as the final payoff minus the cost of setting up the position, of a bull spread position as a function of the underlying asset value. The left hand plot shows a bull spread constructed with 2 call options, whereas the right hand plot shows a bull spread constructed with 2 put options.

However, long positions in the underlying or for that sake a call option is also a bet that the underlying will increase. Thus, one might wonder why anybody would ever consider implementing a bull spread with say two call options. The answer to this question is that it costs less to implement the bull spread strategy than a strategy which is long only the call option. To be specific, the bull spread involves a short position in an out of the money call option. That is with the bull spread the investor collects, up front, the premium for this option.

Ok, that is all nice but why then would anybody consider going long the call without collecting the premium on the shorted option? Again the answer is simple. The reason is that the bull spread has less upside potential as it is capped above because of the shorted call. Again, nothing comes for free in the financial markets and if you want less costs of setting up the portfolio you have to pay by having less upside potential. Thus, when implementing a bull spread these two effects in general have to be balanced.

However, in some situations we may have a strong indication that the maximum the stock will move within the time to maturity is capped above. Remember the technical analysis! Sometimes the outcome of such an analysis is a specific target value which the asset is supposed to reach. If this is the case there is no need to have the full upside potential from long positions in the underlying or in a call option alone. Instead it may be optimal for the investor believing in the technical analysis to implement a bull spread with appropriately chosen strike prices for the used options. In particular, this strategy may well be the one that maximizes profits.

Bear spreads:

A bear spread may be constructed in two different ways. The position may be short a call and long a call with higher strike price, or short a put and long a put with higher strike price. That is a bear spread is essentially a short bull spread. Thus, these types of strategies, because they involve taking opposite position in the same type of option, does not have the potential for large downside risk that the straddles or strangles had.

Because the bear spread is simply a short position in a bull spread the payoff at maturity less the initial costs of the position is the 180 degree horizontally flipped mirror image of that shown in Figure 4. Thus is should be clear that this strategy is a bet that the price of the underlying will decrease. Again, the strategy may be implemented at a lower cost than simple being long the put option, but this comes at the cost of having less upside potential if the asset drops with a large amount.

Butterfly spreads:

What if you believe that the price of the underlying will remain stable? But yet don't want to risk your pants by implementing a short straddle. The answer is to implement a butterfly spread. This strategy involves a position which is long 2 calls with different strike price and short two calls with equal strike price, and with a strike price in between. The payoff at maturity less the initial costs of the position is shown in Figure 5. Alternatively, a similar spread may be constructed with put options.

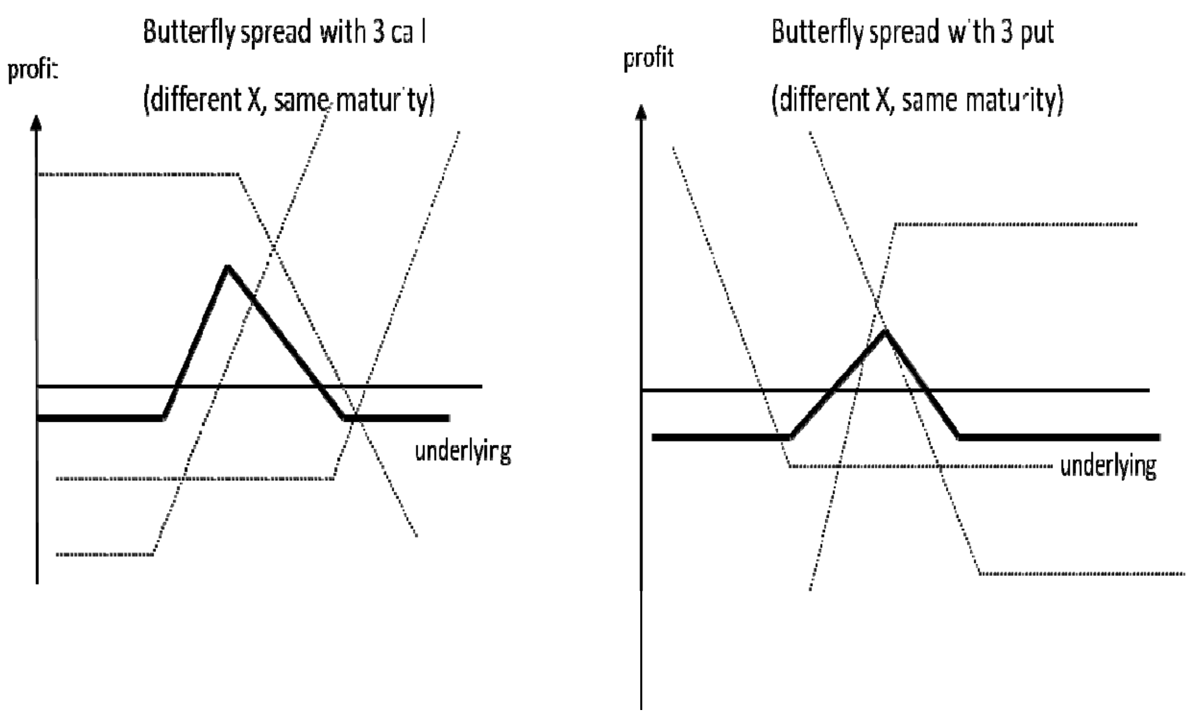

Figure 5: Profit at maturity, measured as the final payoff minus the cost of setting up the position, of a butterfly spread position as a function of the underlying asset value. The left hand plot shows a butterfly spread constructed with 4 call options, whereas the right hand plot shows a butterfly spread constructed with 4 put options.

Calendar spread:

The usual calendar spread, also called a time spread or horizontal spread, involves the purchase of futures/options with a given strike price expiring in a more distant month and the sale of futures/options having the same strike price but which expire in a more nearby month.

The calendar spread is a strategy used by the trader in an attempt to take advantage of a difference in the implied volatilities between two different months' options. The trader will ordinarily implement this strategy when the options he or she is buying have a distinctly lower implied volatility than the options he is writing (selling).

In the typical version of this strategy, a rise in the overall implied volatility of a market's options during the trade will tend very strongly to be to the trader's advantage, and a decline in implied volatility will tend strongly to work to the trader's disadvantage.

If the trader, instead, buys a nearby month's options in some underlying market and sells that same underlying market's further-out options of the same striking price, this is known as a reverse calendar spread. This strategy will tend strongly to benefit from a decline in the overall implied volatility of that market's options over time.

Diagonal spreads:

The diagonal spread is an option spread strategy that involves the simultaneous purchase and sale of equal number of options of the same class, same underlying security but with different strike prices and different expiration months.

The diagonal spread is very much like the calendar spread, where near term options are sold while long term options are bought to take advantage of the rapid time decay in options that are soon to expire.

The main difference between the calendar spread and the diagonal spread lies in the near term outlook. The employer of the diagonal spread has a near term outlook that is slightly more bullish or bearish.

Summary

When trading derivatives, options in particular, exposure may be obtained to a number of factors. The most important factor is exposure to the underlying, so called directional exposure. This allows investors to use derivatives to hedge positions in the underlying, to speculate in the changes of the underlying, or to exploit arbitrage possibilities.

However, as we have seen options may be used to speculate in other factors than just the directional move of the underlying. In particular, it is possible to obtain exposure to the volatility of the underlying. Finally, options can be used to buy time in a special sense!

References

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