Amity School of Business

BBA, Semester

Financial Derivative

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Forwards and futures

Hedging using index futures

To hedge the risk in a portfolio the number of contracts that should be shorted is

$$\beta \frac{P}{F}$$

where P is the value of the portfolio, β is its beta, and F is the value of one futures contract

Example

S&P 500 futures price is 1,000

Value of Portfolio is $5 million

Beta of portfolio is 1.5

Lot size of future contract is 250 indexes

What position in futures contracts on the S&P 500 is necessary to hedge the portfolio?

Changing Beta

- What position is necessary to reduce the beta of the portfolio to 0.75?
- What position is necessary to increase the beta of the portfolio to 2.0?

Problem

S&P 500 index price is $1,000

Value of Portfolio is $5 million

Beta of portfolio is 1.5

Lot size of future contract is 250 indexes

Dividend yield on index = 1% p.a.

Risk free interest rate = 4% p.a.

Problem

- Future contract on the S&P with 4 months of maturity is used to hedge the value of the portfolio over the next 3 months.
- Current contract price = $1010
- Future spot price = $900
- Future contract price = $902

- Derive the payoff from the problem at the end of three month.

Rolling the hedge forward

- We can use a series of futures contracts to increase the life of a hedge
- Each time we switch from one futures contract to another we incur a type of basis risk

Find optimal hedge ratio

Month	Change in Spot price	Change in Future prices
1	0.5	0.56
2	0.61	0.63
3	-0.22	-0.12
4	-0.35	-0.44
5	0.79	0.60
6	0.04	-0.06
7	0.15	0.01
8	0.70	0.80
9	-0.51	-0.56
10	-0.41	-0.46

Problem

- On July 1, an investor holds 50,000 shares of a certain stocks. The market price is $30 per share. The investor is interested in hedging against the movement over the next month. He decided to use September mini S&P futures contract. The index is currently traded at 1,500 and on contract is for $50 times the index. The beta of the stock is 1.3 . What strategy should the investor follow. If he expects a bear and if he expects a bull.

Problem

S&P 500 index price is $1,500

Value of Portfolio is 1,500,000

Beta of portfolio is 1.3

Lot size of future contract is 50 indexes

Dividend yield on index = 4% p.a.

Risk free interest rate = 8% p.a.

Problem

- Future contract on the S&P with 4 months of maturity is used to hedge the value of the portfolio over the next 3 months.
- Current contract price = $1560
- Case 1:-
  - – Future spot price = $1400
  - – Future contract price = $1405
- Case2:-
  - – Future Spot Price = $ 1600
  - – Future Contract Price = 1604

Interest Rates

Types of rates

- Treasury rates
- LIBOR rates
- Repo rates

Continuous compounding

- In the limit as we compound more and more frequently we obtain continuously compounded interest rates
- \$100 grows to $\$100e^{RT}$ when invested at a continuously compounded rate R for time T
- \$100 received at time T discounts to $\$100e^{-RT}$ at time zero when the continuously compounded discount rate is R

Example

- A sum deposited for different period will reap different value

Compounding frequency	value of Rs 100
Annually (m=1)	110
Semi (m=2)	110.25
Qyaterly(m=4)	110.38
Monthly (m = 12)	110.47
Weekly (m = 52)	110.51
Daily (M = 365)	110.52

But when we talk about continuous compounding same investment for a year the result will be

$$100 * e^{0.1} = 110.52$$

Conversion formula

Define

R_c : continuously compounded rate per year

R_m: same rate with compounding *m* times per year

Y = ln x   then x = e^y

$$R_c = m \ln\left(1 + \frac{R_m}{m}\right)$$

$$R_m = m\left(e^{R_c/m} - 1\right)$$

- Consider an effective interest rate quoted at 10% per annum with semi annual compounding. Find R_c and R_m

Zero rates

A zero rate (or spot rate), for maturity T is the rate of interest earned on an investment that provides a payoff only at time T

Example

Maturity (years)	Rate (% cont comp)
0.5	5.0
1.0	5.8
1.5	6.4
2.0	6.8

- To calculate the cash price of a bond we discount each cash flow at the appropriate zero rate
- In our example, the theoretical price of a two-year bond providing a 6% coupon semiannually
- FV = 100

Bond pricing

- To calculate the cash price of a bond we discount each cash flow at the appropriate zero rate
- In our example, the theoretical price of a two-year bond providing a 6% coupon semiannually is

$$3e^{-0.05\times0.5} + 3e^{-0.058\times1.0} + 3e^{-0.064\times1.5}$$
$$+ 103e^{-0.068\times2.0} = 98.39$$

Bond Yield

- The bond yield is the discount rate that makes the present value of the cash flows on the bond equal to the market price of the bond
- Suppose that the market price of the bond in our example equals its theoretical price of 98.39
- The bond yield (continuously compounded) is given by solving

$$3e^{-y\times 0.5} + 3e^{-y\times 1.0} + 3e^{-y\times 1.5} + 103e^{-y\times 2.0} = 98.39$$

to get y=0.0676 or 6.76%.

Par Yield

- The par yield for a certain maturity is the coupon rate that causes the bond price to equal its face value.
- In our example we solve

$$\frac{c}{2}e^{-0.05\times 0.5}+\frac{c}{2}e^{-0.058\times 1.0}+\frac{c}{2}e^{-0.064\times 1.5}$$

$$+\left(100+\frac{c}{2}\right)e^{-0.068\times 2.0}=100$$

$$\text{to get } c = 6.87$$

Par Yield

In general if m is the number of coupon payments per year, P is the present value of \$1 received at maturity and A is the present value of an annuity of \$1 on each coupon date

$$c = \frac{(100 - 100P)m}{A}$$

Bootstrap method

- Treasury zero rate can be calculated by using boot strap method.

Sample data

Bond Principal (dollars)	Time to Maturity (years)	Annual Coupon (dollars)	Bond Cash Price (dollars)
100	0.25	0	97.5
100	0.50	0	94.9
100	1.00	0	90.0
100	1.50	8	96.0
100	2.00	12	101.6

Bootstrap method

- An amount 2.5 can be earned on 97.5 during 3 months.
- The 3-month rate is 4 times 2.5/97.5 or 10.256% with quarterly compounding
- Similarly the 6 month and 1 year rates are 10.469% and 10.536% with continuous compounding

Bootstrap method

- To calculate the 1.5 year rate we solve

$$4e^{-0.10469\times 0.5} + 4e^{-0.10536\times 1.0} + 104e^{-R\times 1.5} = 96$$

to get R = 0.10681 or 10.681%

- Similarly the two-year rate is 10.808%

Valuing futures and forwards

Consumption vs Investment assets

- Investment assets are assets held by significant numbers of people purely for investment purposes (Examples: gold, silver)
- Consumption assets are assets held primarily for consumption (Examples: copper, oil)

Assumptions

- The markets are perfect
- There is no transaction cost
- All the assets are infinitely divisible
- Bid-ask spread do not exists
- There is no restriction on short selling.

Short Selling

- Short selling involves selling securities you do not own
- Your broker borrows the securities from another client and sells them in the market in the usual way

Short Selling

- At some stage you must buy the securities back so they can be replaced in the account of the client
- You must pay dividends and other benefits the owner of the securities receives

Notation for valuing futures and forwards contracts

S_0: Spot price today

F_0: Futures or forward price today

T: Time until delivery date

r: Risk-free interest rate for maturity T

An Arbitrage opportunity

- Suppose that:
  - The spot price of a non-dividend paying stock is $40
  - The 3-month forward price is $43
  - The 3-month US$ interest rate is 5% per annum
- Is there an arbitrage opportunity?

An Arbitrage opportunity

- Suppose that:
  - The spot price of non-dividend paying stock is $43
  - The 3-month forward price is US$39
  - The 1-year US$ interest rate is 5% per annum
- Is there an arbitrage opportunity?

Carry Pricing Model

- Model stipulates hat the forward of future prices defined as the value of one unit of the asset underlying the contract is equal to the sum of the spot price and the carrying cost incurred by buying and holding on the asset less the carry return is any.

Price = spot price + carry cost – carry return

The forward Prices

If the spot price of an investment asset is S_0 and the futures price for a contract deliverable in T years is F_0, then

$$F_0 = S_0 e^{rT}$$

where r is the 1-year risk-free rate of interest.

In our examples, S_0 =40, T=0.25, and r=0.05 so that

$$F_0 = 40e^{0.05\times 0.25} = 40.50$$

Problem

- A forward contract on a dividend paying share which is available at Rs70 , to mature in 3-months time. If the risk free rate of interest be 8% per annum compounded continuously the contract should be priced at -

If Short sales is not possible

Formula still works for an investment asset because investors who hold the asset will sell it and buy forward contracts when the forward price is too low

When an Investment Asset Provides a Known Income

$$F_0 = (S_0 - I\,)e^{rT}$$

where I is the present value of the income during life of forward contract

Arbitrage opportunity happens if

$$F_0 > (S_0 - I\,)e^{rT}$$

$$F_0 < (S_0 - I\,)e^{rT}$$

Problem

- 6 months forward contract on 100 shares with a price of rs 38 each. The risk free rate of interest is 10%p.a.(Cc) the share provides yield of Rs 1.5 per 4 months. Determine the value of the forward contract.

When an investment asset provides a known yield

$$F_0 = S_0 e^{(r-q)T}$$

where q is the average yield during the life of the contract (expressed with continuous compounding)

Problem

- A stock underlying the idex provides a dividend yield of 4% per annum, the current value of the index is 520 and that the continuous compounded risk free rate of interest is 10% per annum. Find the value of 3 month forward contract

Valuing a forward contract

- Suppose that

  K is delivery price in a forward contract and

  F_0 is forward price that would apply to the contract today

- The value of a long forward contract, f, is

$$f = (F_0 - K)e^{-rT}$$

- Similarly, the value of a short forward contract is

$$(K - F_0)e^{-rT}$$

Forwards Vs Future

- Forward and futures prices are usually assumed to be the same. When interest rates are uncertain they are, in theory, slightly different:
- A strong positive correlation between interest rates and the asset price implies the futures price is slightly higher than the forward price
- A strong negative correlation implies the reverse

Stock Index

- Can be viewed as an investment asset paying a dividend yield
- The futures price and spot price relationship is therefore

$$F_0 = S_0\, e^{(r-q)T}$$

where q is the average dividend yield on the portfolio represented by the index during life of contract

Stock Index

- For the formula to be true it is important that the index represent an investment asset
- In other words, changes in the index must correspond to changes in the value of a tradable portfolio
- The Nikkei index viewed as a dollar number does not represent an investment asset

Index arbitrage

- When $F_0 > S_0e^{(r-q)T}$ an arbitrageur buys the stocks underlying the index and sells futures
- When $F_0 < S_0e^{(r-q)T}$ an arbitrageur buys futures and shorts or sells the stocks underlying the index

Index arbitrage

- Index arbitrage involves simultaneous trades in futures and many different stocks
- Very often a computer is used to generate the trades
- Occasionally simultaneous trades are not possible and the theoretical no-arbitrage relationship between F_0 and S_0 does not hold

Futures and forwards of currencies

- A foreign currency is analogous to a security providing a dividend yield
- The continuous dividend yield is the foreign risk-free interest rate
- It follows that if r_f is the foreign risk-free interest rate

$$F_0 = S_0 e^{(r - r_f)T}$$

Futures on consumption asset

$$F_0 \leq S_0 e^{(r+u)T}$$

where u is the storage cost per unit time as a percent of the asset value.

Alternatively,

$$F_0 \leq (S_0 + U)e^{rT}$$

where U is the present value of the storage costs.

The cost of carry

- The cost of carry, c, is the storage cost plus the interest costs less the income earned
- For an investment asset $F_0 = S_0 e^{cT}$
- For a consumption asset $F_0 \leq S_0 e^{cT}$
- The convenience yield on the consumption asset, y, is defined so that
  $F_0 = S_0 \, e^{(c-y)T}$

Future price and expected future spot prices

- Suppose k is the expected return required by investors on an asset
- We can invest $F_0 e^{-rT}$ at the risk-free rate and enter into a long futures contract so that there is a cash inflow of S_T at maturity
- This shows that

$$F_0 e^{-rT} e^{kT} = E(S_T)$$

or

$$F_0 = E(S_T) e^{(r-k)T}$$

Future price and expected future spot prices

- If the asset has
  - no systematic risk, then $k = r$ and F_0 is an unbiased estimate of S_T
  - positive systematic risk, then $k > r$ and $F_0 < E\,(S_T)$
  - negative systematic risk, then $k < r$ and $F_0 > E\,(S_T)$