Amity School of Business

BBA, Semester V

Priyank Badola

Options

- A call option is an option to buy a certain asset by a certain date for a certain price (the strike price)
- A put option is an option to sell a certain asset by a certain date for a certain price (the strike price)

American and European Options

- An American option can be exercised at any time during its life
- A European option can be exercised only at maturity

Options position

- Long call
- Long put
- Short call
- Short put

Exchange trading Options

- Chicago Board Options Exchange
- American Stock Exchange
- Philadelphia Stock Exchange
- Pacific Exchange
- LIFFE (London)
- Eurex (Europe)
- and many more (see list at end of book)

Types of traders

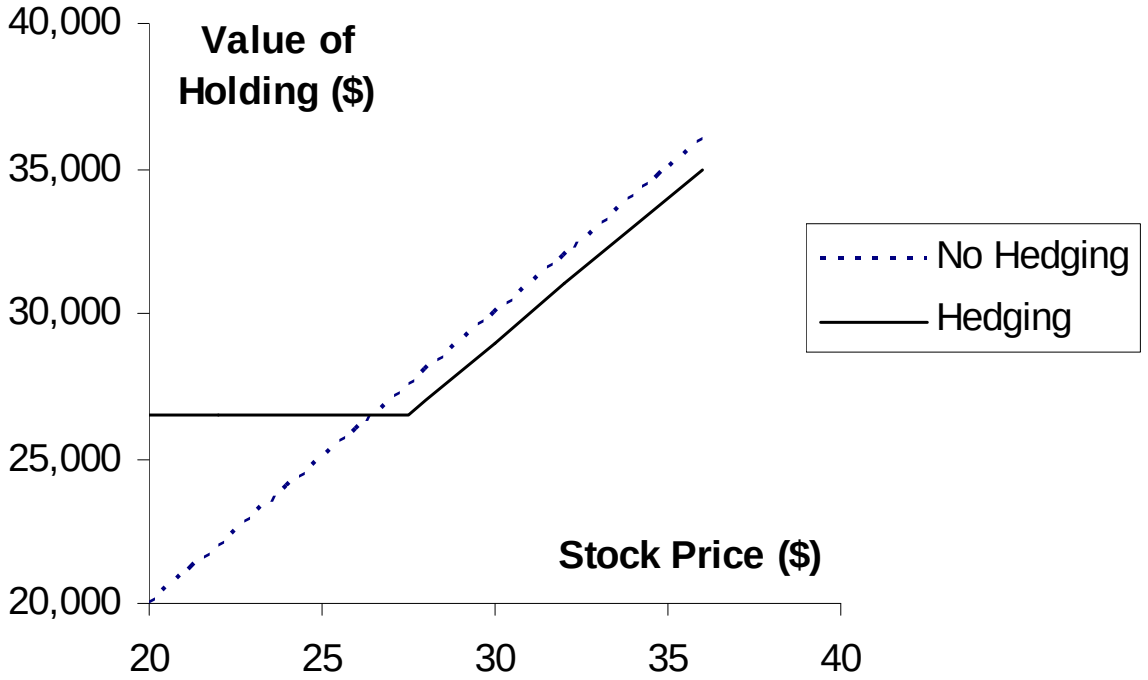
- Hedgers
- Speculators
- Arbitrageurs

Some of the largest trading losses in derivatives have occurred because individuals who had a mandate to be hedgers or arbitrageurs switched to being speculators (See for example Barings Bank, Business Snapshot 1.2, page 15)

Hedging example

- An investor owns 1,000 Microsoft shares currently worth $28 per share. A two-month put with a strike price of $27.50 costs $1. The investor decides to hedge by buying 10 contracts

Value of Microsoft Shares with and without Hedging Options



Speculation example

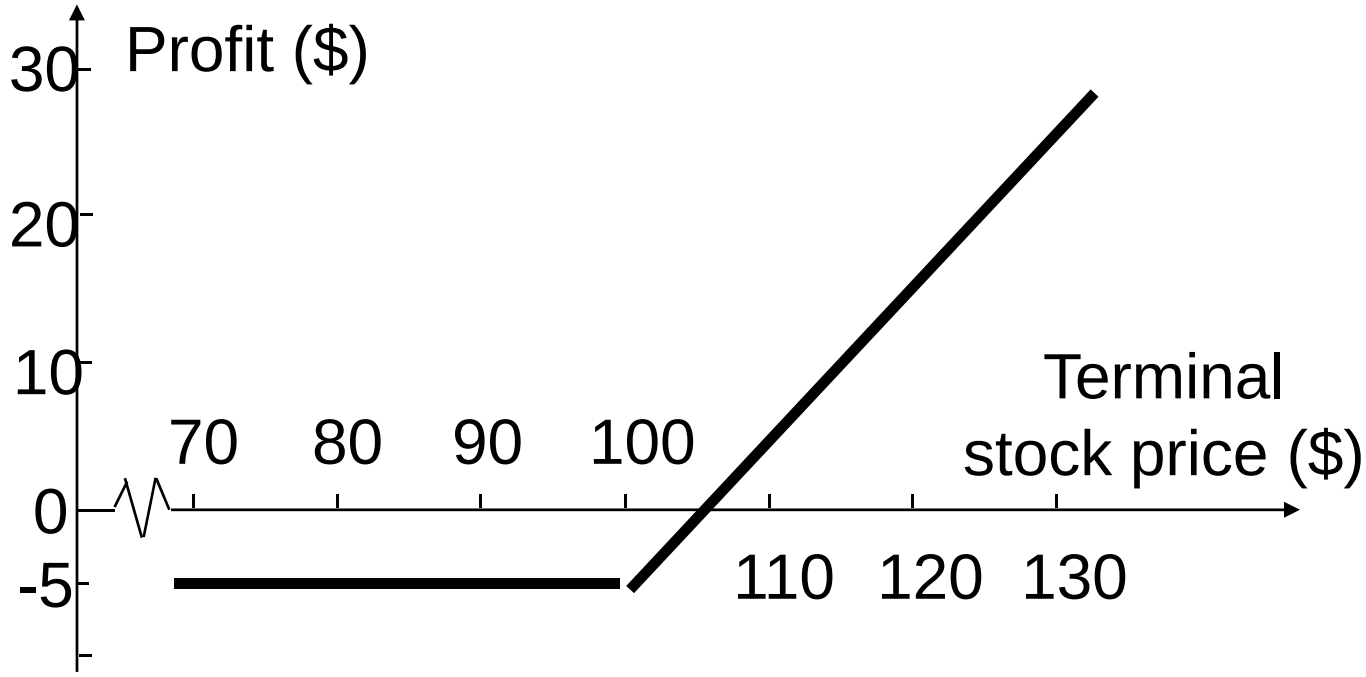
- An investor with $4,000 to invest feels that Amazon.com's stock price will increase over the next 2 months. The current stock price is $40 and the price of a 2-month call option with a strike of 45 is $2
- What are the alternative strategies?

Hedge funds

- Hedge funds are not subject to the same rules as mutual funds and cannot offer their securities publicly.
- Mutual funds must
  - disclose investment policies,
  - makes shares redeemable at any time,
  - limit use of leverage
  - take no short positions.
- Hedge funds are not subject to these constraints.
- Hedge funds use complex trading strategies are big users of derivatives for hedging, speculation and arbitrage

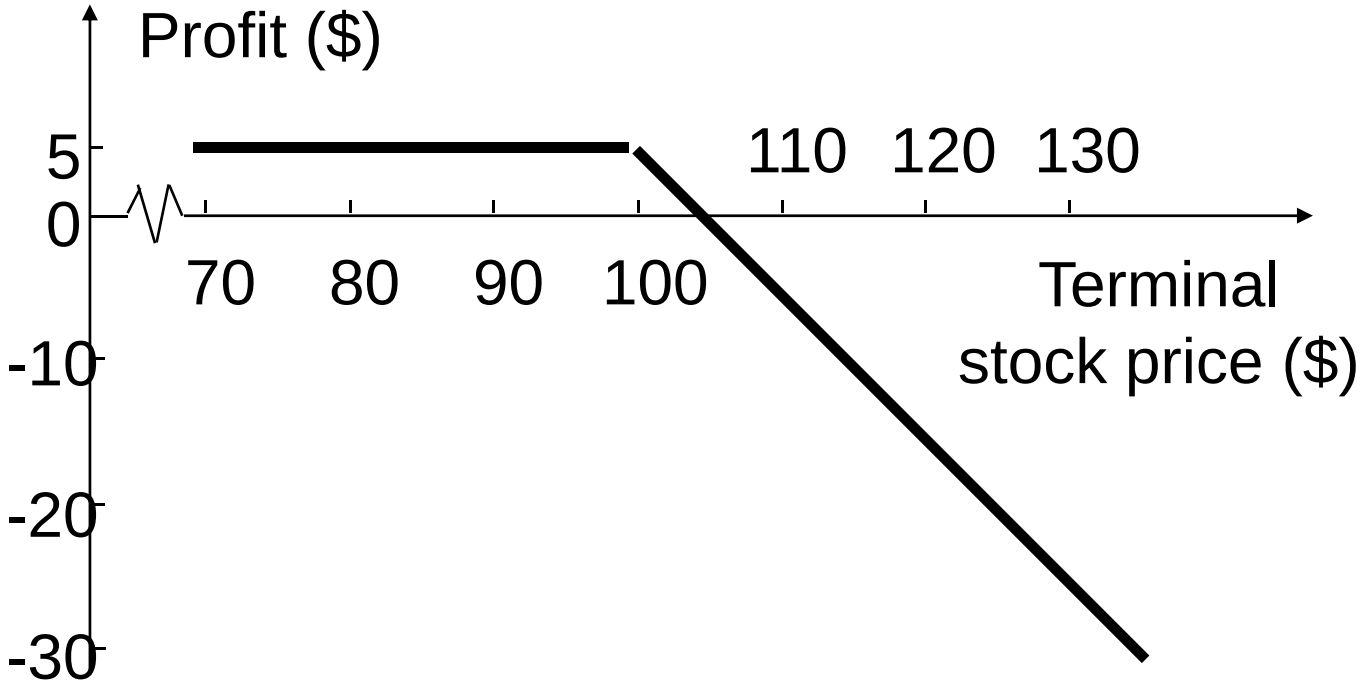
Long Call

Profit from buying one European call option: option price = $5, strike price = $100, option life = 2 months



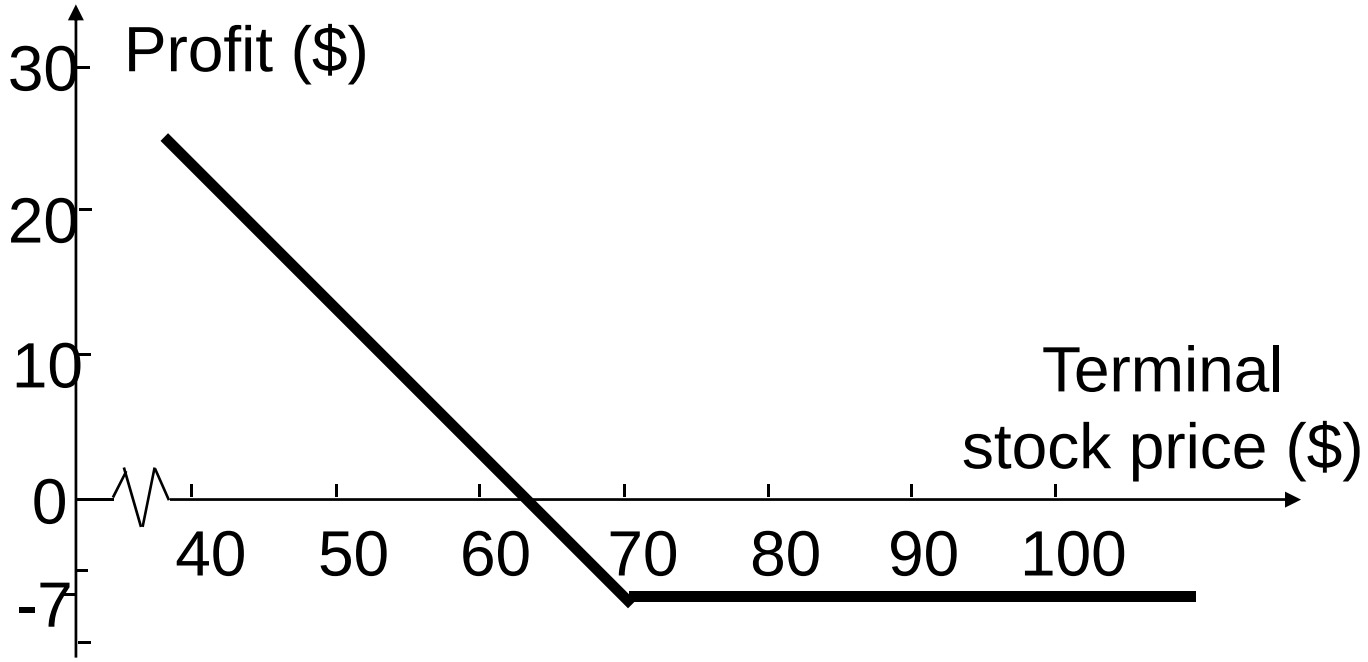
Short Call

Profit from writing one European call option: option price = $5, strike price = $100



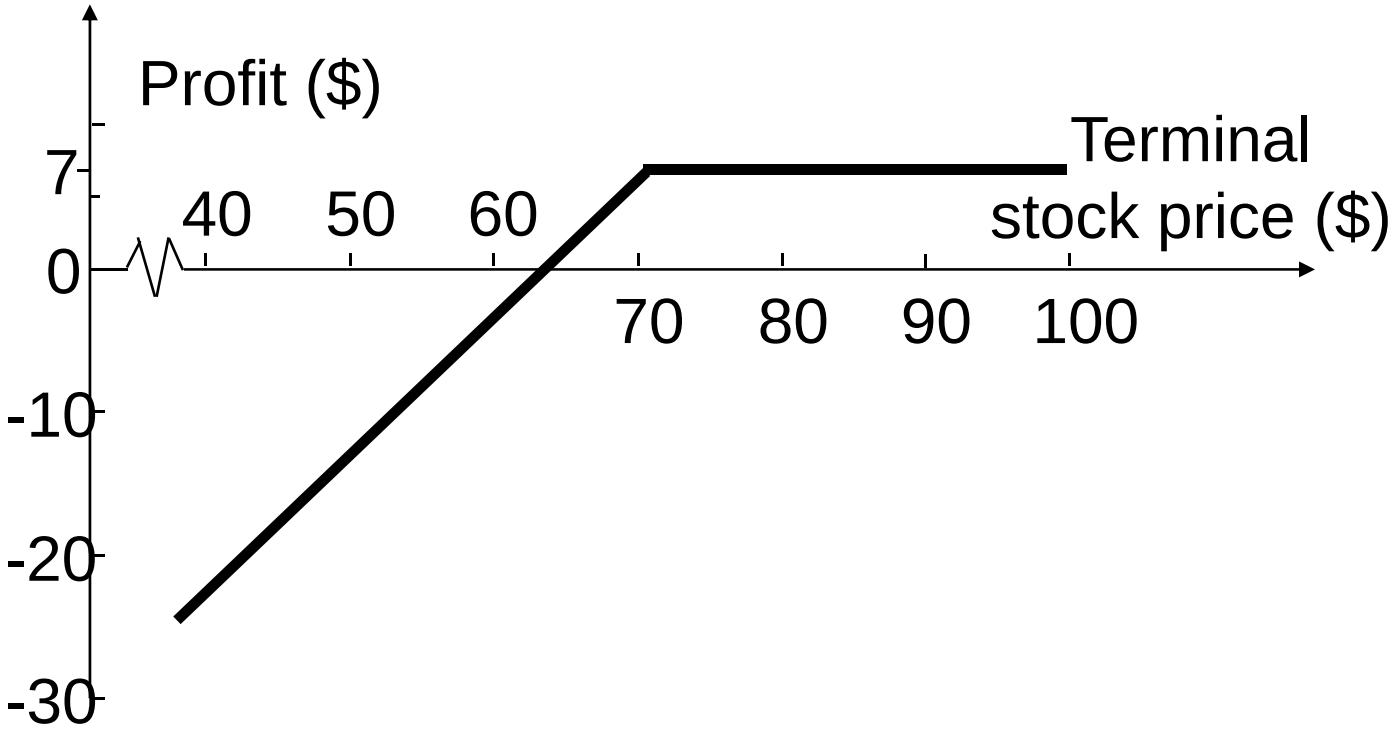
Long Put

Profit from buying a European put option: option price = $7, strike price = $70



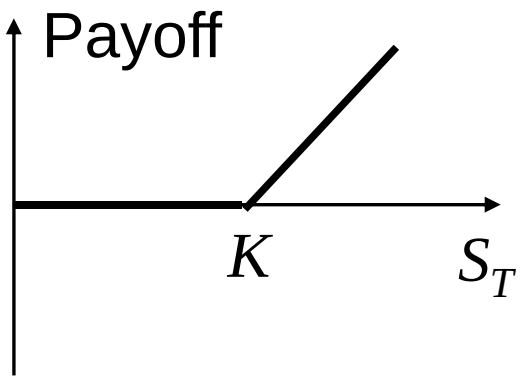
Short Call

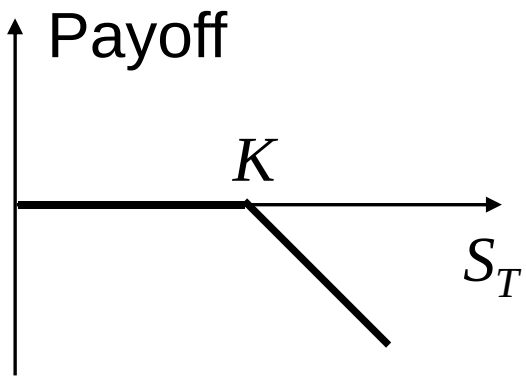
Profit from writing a European put option: option price = $7, strike price = $70

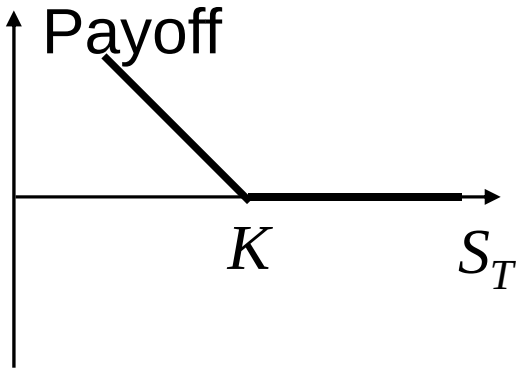


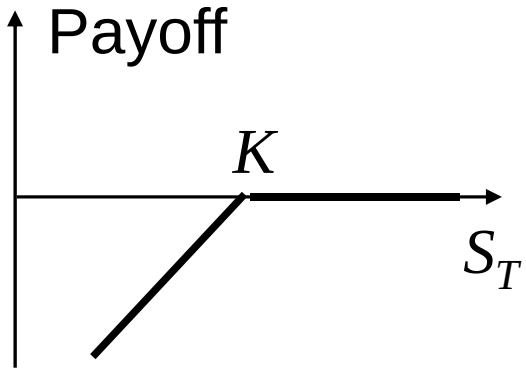
Payoffs from Options-What is the Option Position in Each Case?

K = Strike price, S_T = Price of asset at maturity









Assets Underlying Exchange Traded Options - Short Call

- Stocks
- Foreign Currency
- Stock Indices
- Futures

Specification of Exchange Traded Options

- Expiration date
- Strike price
- European or American
- Call or Put (option class)

Terminology

Moneyness :

- At-the-money option
  - c : $S = K$ p : $S = K$
- In-the-money option
  - c : $S > K$ p : $S < K$
- Out-of-the-money option
  - c : $S < K$ p : $S > K$

Terminology

- Option class
- Option series
- Intrinsic value c: max(S-K,0) p:max(K-S,0)
- Time value = option value – intrinsic value
- Position limits
- Exercise limit.
- Offsetting orders

Adjustment

- Dividends
- Stock splits

  Suppose A call option to buy 100 shares of a company for $30 per share. Company makes a 2 for 1 stock split the term and condition will be changed in a manner.

  Suppose a company declares a cash dividend of Rs13.50 per common share and the market price is Rs60.

Market makers

- Most exchanges use market makers to facilitate options trading
- A market maker quotes both bid and ask prices when requested
- The market maker does not know whether the individual requesting the quotes wants to buy or sell

Warrants

- Warrants are options that are issued by a corporation or a financial institution
- The number of warrants outstanding is determined by the size of the original issue and changes only when they are exercised or when they expire

Warrants

- The issuer settles up with the holder when a warrant is exercised
- When call warrants are issued by a corporation on its own stock, exercise will usually lead to new treasury stock being issued

Executive stock options

- Executive stock options are a form of remuneration issued by a company to its executives
- They are usually at the money when issued
- When options are exercised the company issues more stock and sells it to the option holder for the strike price

Executive stock options

- They become vested after a period of time (usually 1 to 4 years)
- They cannot be sold
- They often last for as long as 10 or 15 years
- Accounting standards now require the expensing of executive stock options

Convertible bonds

- Convertible bonds are regular bonds that can be exchanged for equity at certain times in the future according to a predetermined exchange ratio
- Very often a convertible is callable
- The call provision is a way in which the issuer can force conversion at a time earlier than the holder might otherwise choose

Option Pricing: Binomial Model

Notations

- c : European call option price
- p : European put option price
- S_0 : Stock price today
- K : Strike price
- T : Life of option
- σ: Volatility of stock price
- C : American Call option price
- P : American Put option price
- S_T :Stock price at option maturity
- D : Present value of dividends during option's life
- r : Risk-free rate for maturity T with cont comp

Effect of variable on option pricing

Variable	c	p	C	P
S_0	+	−	+	−
K	−	+	−	+
T	?	?	+	+
σ	+	+	+	+
r	+	−	+	−
D	−	+	−	+

American Vs European options

An American option is worth at least as much as the corresponding European option

$$C \geq c$$

$$P \geq p$$

Calls: An arbitrage opportunity

- Suppose that

  $c = 3$ $\qquad$ $S_0 = 20$

  $T = 1$ $\qquad$ $r = 10\%$

  $K = 18$ $\qquad$ $D = 0$

- Is there an arbitrage opportunity?

Actions

- Short the stock and buy the call
- Cash Flow is 20-3=17
- Invest it @10% for 1 yr
- Cash Flow at the end of 1 yr =17e $^{0.1x1}$

=18.79 (Due to the investment of this cash)

- If the price after a yr is more than 18, then the option is exercised and profit is 0.79
- If the price is less than 18, more profit.

Terminologies

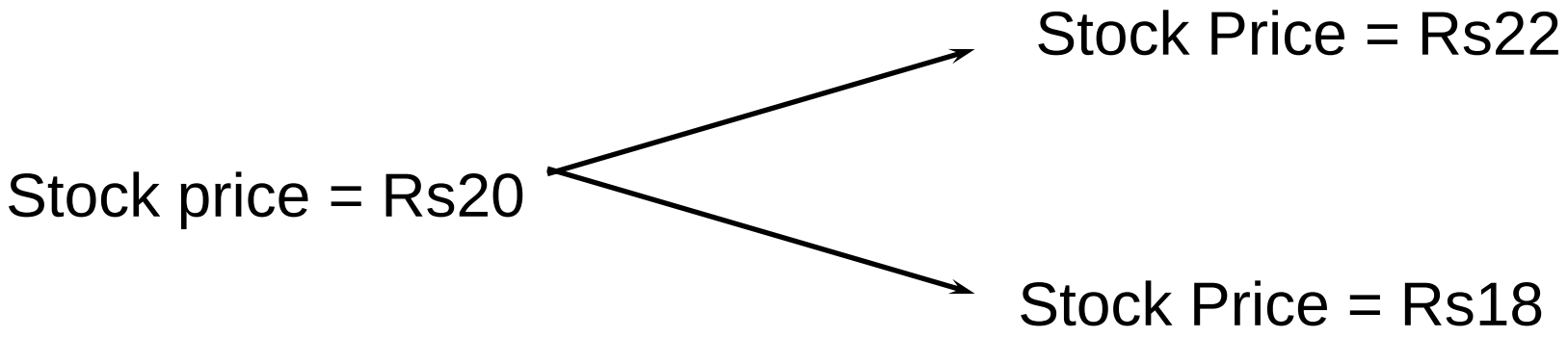
- Upper bound
  - C : stock price
  - P : Strike price (K)
- Lower bound
  - C : $S_0 - Ke^{-rt}$
  - P : $Ke^{-rt} - S_0$

A simple Binomial Model

- Assumptions
  - Arbitrage opportunity does not exists
  - No uncertainity about the value of the portfolio exists after 3 months.

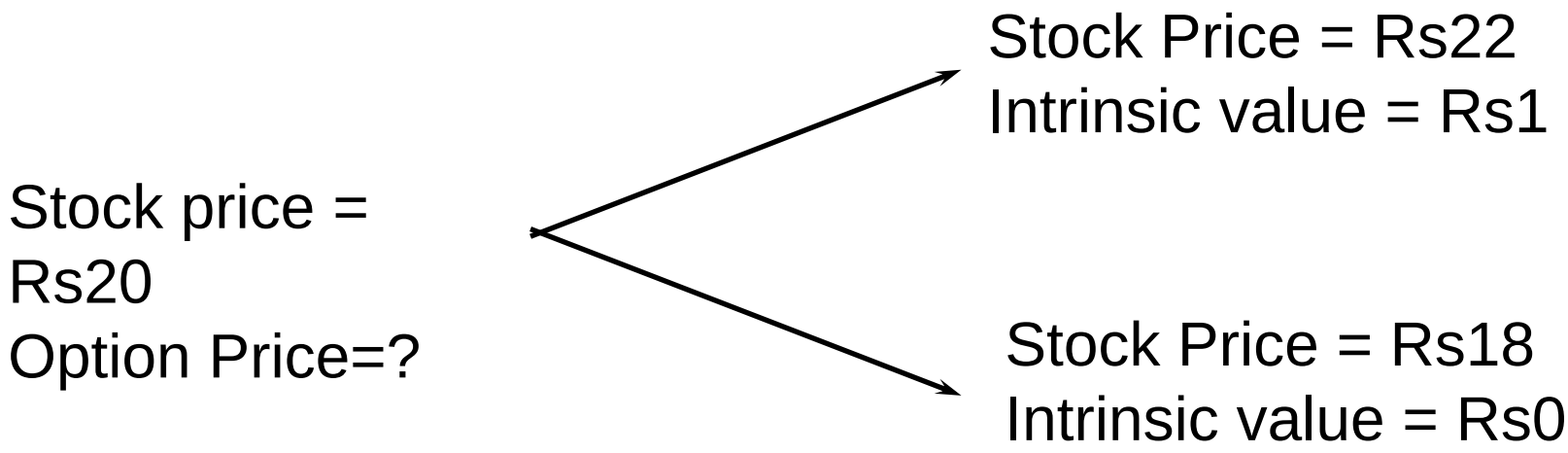
A simple Binomial Model

- A stock price is currently Rs20
- In three months it will be either Rs22 or Rs18



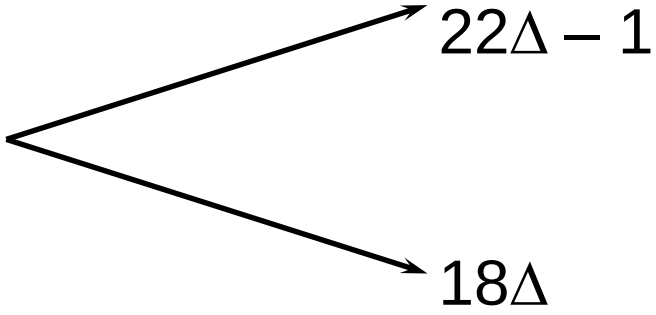
A European call option

A 3-month call option on the stock has a strike price of 21.



Setting up a riskless portfolio

- Consider the Portfolio: long Δ shares
  short 1 call option



- Portfolio is riskless when $22\Delta - 1 = 18\Delta$ or
  $\Delta = 0.25$

Valuing the portfolio

(Risk-Free Rate is 12%)

- The riskless portfolio is:

  long 0.25 shares
  short 1 call option

- The value of the portfolio in 3 months is

$$22 \times 0.25 - 1 = 4.50$$

- The value of the portfolio today is

$$4.5e^{-0.12 \times 0.25} = 4.3670$$

Valuing the option

- The portfolio that is

  long 0.25 shares
  short 1 option

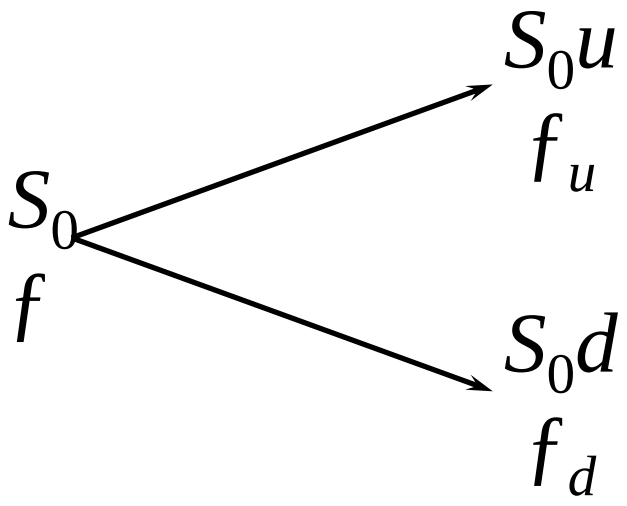
  is worth 4.367
- The value of the shares is

  5.000 (= 0.25×20)
- The value of the option is therefore

  0.633 (= 5.000 – 4.367)

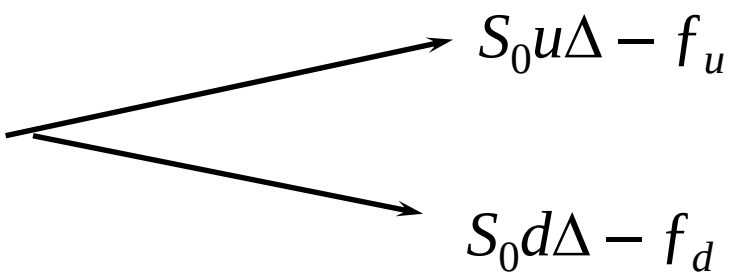
Generalisation

A derivative lasts for time T and is dependent on a stock



Generalisation

- Consider the portfolio that is long Δ shares and short 1 derivative



- The portfolio is riskless when $S_0u\Delta - f_u = S_0d\Delta - f_d$ or

$$\Delta = \frac{f_u - f_d}{S_0u - S_0d}$$

Generalisation

- Value of the portfolio at time T is
$$S_0 u\Delta - f_u$$
- Value of the portfolio today is
$$(S_0 u\Delta - f_u)e^{-rT}$$
- Another expression for the portfolio value today is $S_0\Delta - f$
- Hence
$$f = S_0\Delta - (S_0 u\Delta - f_u)e^{-rT}$$

Generalisation

- Substituting for Δ we obtain

$$f = [\, pf_u + (1-p)f_d \,]e^{-rT}$$

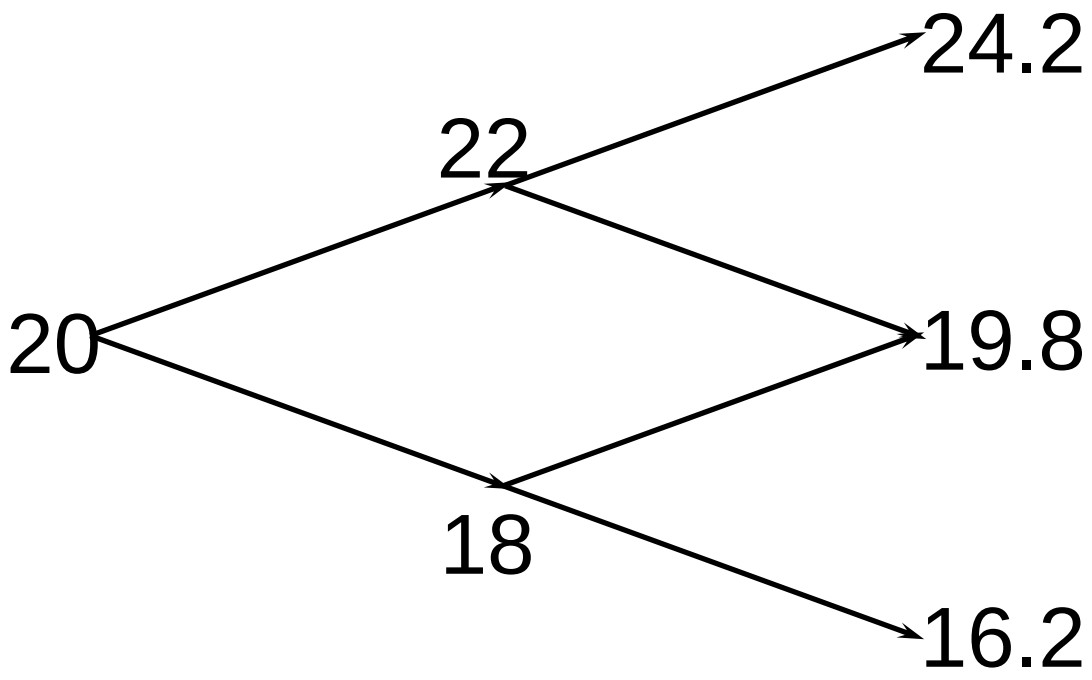
where

$$p = \frac{e^{rT} - d}{u - d}$$

Irrelavence of stock's expected returns

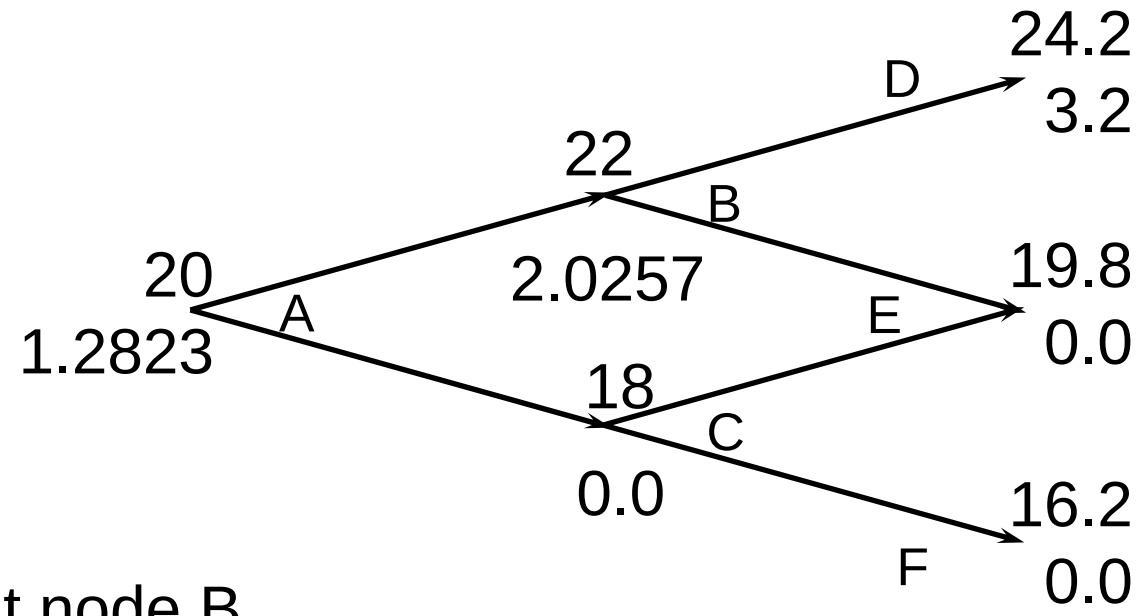
- When we are valuing an option in terms of the the price of the underlying asset, the probability of up and down movements in the real world are irrelevant
- This is an example of a more general result stating that the expected return on the underlying asset in the real world is irrelevant

A two step example



- Each time step is 3 months
- K=21, r=12%

Valuing a call option



- Value at node B

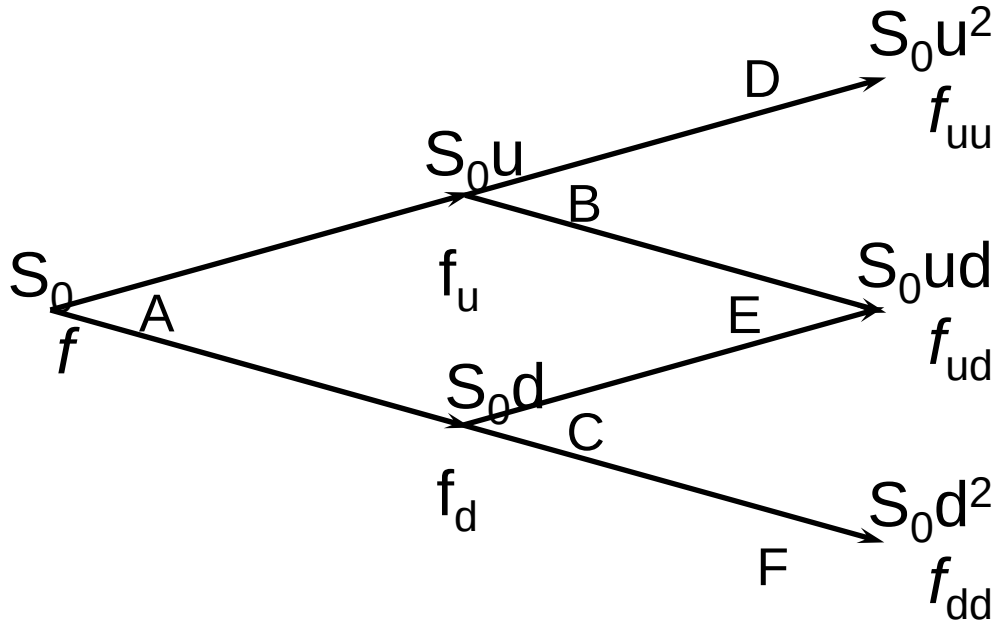
  $= e^{-0.12\times0.25}(0.6523\times3.2 + 0.3477\times0) = 2.0257$

- Value at node A

  $= e^{-0.12\times0.25}(0.6523\times2.0257 + 0.3477\times0)$

  $= 1.2823$

Valuing a call option

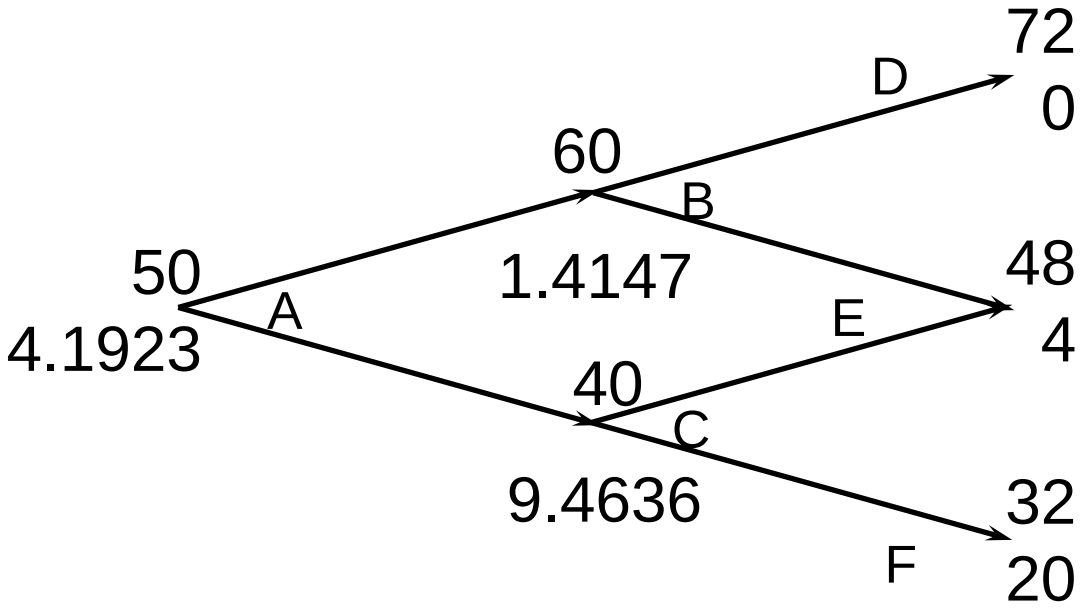


$$f = [\, p^2 f_{uu} + 2p(1-p)\, f_{ud} + (1-p)^2 f_{dd}]e^{-2rT}$$

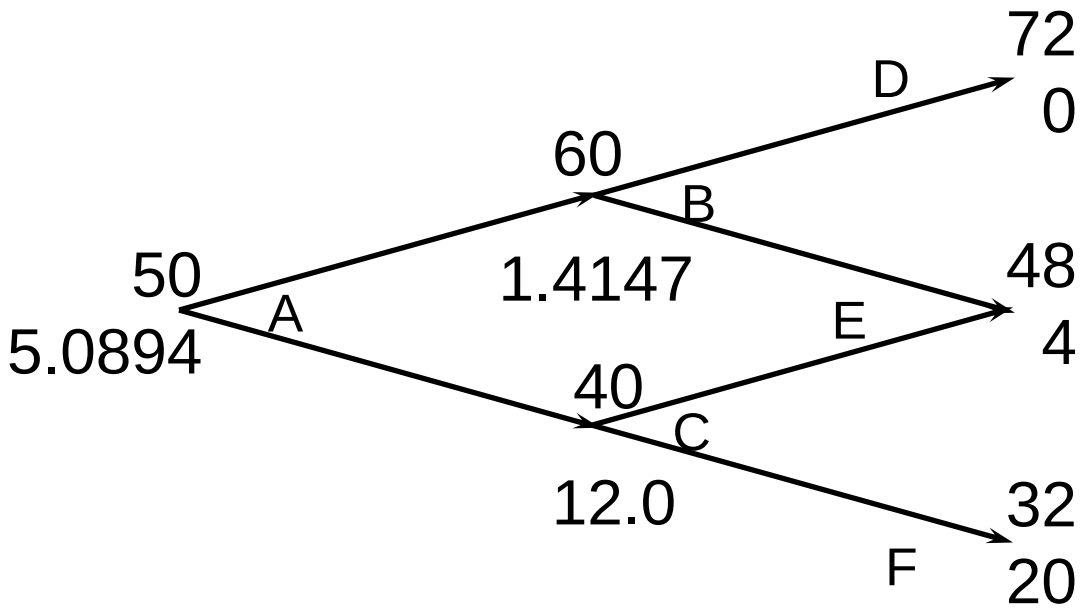
Valuing a put option

K = 52, time step = 1yr

r = 5%



Valuing a American put option



The Black-Scholes-Merton Model

- Before Black and Scholes, there was a widespread belief that the expected growth of the underlying ought to affect the option price.
- Black and Scholes demonstrate that this is not true. It tells us what is important and what is not
- It doesn’t promise to produce the exact prices that show up in the market, but certainly does a remarkable job of pricing options within the framework of assumptions of the model.

- It works on the equation called *Geometric Brownian Motion* - lognormal distribution that the logarithm of the stock's return will follow the normal (bell shaped) distribution.
- option's price is determined by
  - time and the
  - underlying stock price.
- Factors do affect the price of option but not allowed to change
  - the volatility
  - the exercise price
  - the risk–free rate

- By forming a portfolio consisting of a long position in stock and a short position in calls, the risk of the stock is eliminated. This hedged portfolio is obtained by setting the number of shares of stock equal to the approximate change in the call price for a change in the stock price. This mix of stock and calls must be revised continuously, a process known as delta hedging

- It uses stochastic calculus to which defines how the option price changes in terms of the change in the stock price and time to expiration. This hedged combination should grow in value at the risk–free rate.
- The result then is a partial differential equation.
- The solution is found by forcing a condition called a boundary condition which makes the option price to converge to the exercise value at expiration.

The stock price assumption

- Consider a stock whose price is S
- In a short period of time of length Δt, the return on the stock is normally distributed:

$$\frac{\Delta S}{S} \approx \phi\left(\mu \Delta t, \sigma\sqrt{\Delta t}\right)$$

where μ is expected return and σ is volatility

Assumptions

- Short selling of securities with full use of proceeds is permitted
- There is no security tax or transaction cost.
- The securities are divisible
- There are no dividend during the life of the derivative
- There is no riskless arbitrage opportunity
- Risk free rate is constant for all maturity
- Security trading is continuous

Black Scholes models inputs

- Current prices
- The option's strike price
- Length of time until the option expires worthless
- An estimate of future volatility known as implied volatility
- Risk free rate of return

Problems of Black Scholes

- First, the Black-Scholes Model assumes that the risk-free rate and the stock's volatility are constant.
- Second, the Black-Scholes Model assumes that stock prices are continuous and that large changes (such as those seen after a merger announcement) don't occur.
- Third, the Black-Scholes Model assumes a stock pays no dividends until after expiration.
- Fourth, analysts can only estimate a stock's volatility instead of directly observing it, as they can for the other inputs.
- Fifth, the Black-Scholes Model tends to misprice options that involve high-dividend stocks

$$
\begin{aligned}
C &= SN(d_1) - Xe^{-rT}N(d_2) \\
P &= Xe^{-rT}N(-d_2) - SN(-d_1) \\
\text{where } d_1 &= \frac{\ln \frac{S}{X} + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \\
\text{and } d_2 &= d_1 - \sigma\sqrt{T}
\end{aligned}
$$