Session 4

Practice Questions

Exercise 1: Futures index pricing

Suppose the S&P/TSX60 value today is 900. What is the future index price for delivery on 3 month? Assume a risk-free rate of 2% and a dividend yield of 3% (both interest rates and continuously compounded).

Exercise 2: Marking to Market – Simple Case

A tomato farmer is trying to secure a selling price for his next crop. In the same time, McTonald's is trying to secure a buying price to determine how much to charge for a Big Mac next year. Early September, the farmer and the corporation decide to enter into a futures contract requiring the delivery of 5 tons of tomatoes in December at a price of $3.50 per kg (1 ton = 1000kg).

Suppose the price on futures contracts for tomatoes increases to $4 per kg the following day, then decreases to $3.8 per kg the day after.

1. Who is the holder of the long position? Of the short position?
2. Who gains from the increase in the price of the future contract? From the decrease in the price of the future contract?
3. Illustrate the marking-to-market from the farmer's perspective.

Exercise 3: Marking to market

Suppose you buy a future in British pound at 1.5712 ($/£), which means "going long British pound." The price of the future is 1.5736 at t+1, 1.5710 at t+2, and 1.5675 at t+3. Suppose an initial margin of $2,000, a maintenance margin of $1,500 and a contract multiplier of 62500.

1. When will you marking to market be positive? Negative
2. Illustrate the marking to market to be made between t and t+3.

Exercise 4: Marking to market – Future on a stock

An investor is taking a long position in a future contract to buy ABC shares at 35.25$ per share. Suppose the initial margin required is 2500$ and the maintenance margin is 2000$. A future contract is an agreement to buy/sell 100 units of shares. Complete the following table. Suppose that if a margin call is made, the investor simply adds the minimum amount of money needed.

Day	Future Price	Margin account
April 9	$35.25	$ 2500
April 10	$33.00	

April 11	$34.50	
April 12	$29.50	
April 13	$ 30.00	
April 14	$37.75	

Exercise 5: Hedging a portfolio of stocks

A portfolio manager is currently managing a diversified portfolio comprised of Canadian stocks worth 10 000 000$. He asks you to implement a hedging strategy so that the value of his portfolio is completely hedged during his holidays. After some calculations, you determine the beta of the portfolio to be 1.75. The future contract underlying asset is the S&P/TSX60. The price of the contract today is 900 and the multiplier of the contract is 200.

1. Which position do you have to take in the future market to hedge the portfolio?
2. Compute the optimal number of contract to trade.
3. Suppose after a certain period of time, the portfolio value increased by 18% while the S&P/TSX60 value increased by 13%. Determine the value of the hedged portfolio.
4. Suppose now the portfolio value decreased by 18% while the S&P/TSX60 value decreased by 13%. Determine the value of the hedged portfolio.
5. Why is the value of the hedged portfolio different from 10 000 000 $? Explain.

Exercise 6: Future Arbitrage

1. Suppose ABC share is currently trading at 35.65$. What is the value of a future contract on this share assuming the continuously compounded interest rate is 3%, the dividend yield is 1% and 20 days are left to the expiry of the contract?
2. If the future quoted price is 37$, which arbitrage strategy could you implement to make a certain profit? Explain all the different steps of your reasoning. Suppose that at the expiry of the future contract, the share trades at 30.45? What if the share was trading at 39$?
3. If the future quoted price is 34$, which arbitrage strategy could you implement to make a certain profit? Explain all the different steps of your reasoning. Suppose that at the expiry of the future contract, the share trades at 30.45?

Solution

Exercise 1: Futures index pricing

$F = S_0 * e^{(r-q)*T}$ = 900 * $e^{(0.02-0.03)*(\frac{3}{12})}$ = $ 897.75

Exercise 2: Marking to Market – Simple Case

1. The farmer is the holder of the short position (he has agreed to sell the underlying asset - tomatoes) and McTonald's is the holder of the long position (it has agreed to buy the asset).
2. McTonald's gains from the increase in the price of the future contract.

   The farmer gains from the decrease in the price of the future contract.

3.

Time	Future Price	Marking-to-Market
t	3.5$/kg	0
t+1	4$/kg	(3.5 – 4)*5000 = -2500$
t+2	3.8$/kg	(4 - 3.8)*5000 = 1000$

Exercise 3: Marking to market

1. You will gain from the appreciation of the British pound against the dollar (value superior to 1.5712).

   You will take losses if the British pound depreciates against the dollar (value inferior to 1.5712).

2.

Time	Future Price	Price Change	Gain/Loss	Cumulated Gain/Loss	Margin Account
t	1.5712				2000 $
t+1	1.5736	+ 0.0024 $	+ 150 $	+ 150 $	2150 $
t+2	1.5710	- 0.0026 $	- 162.5 $	- 12.5 $	1987.5 $
t+3	1.5675	- 0.0035 $	- 218.75 $	- 231.25 $	1768.75 $

Exercise 4: Marking to market – Future on a stock

Time	Price	Daily Profit/Loss	Cumulative P&L	Margin	Margin Call
0	35.25	0	0	2500	0
1	33	-225	-225	2275	0
2	34.5	150	-75	2425	0
3	29.5	-500	-575	1925	575
4	30	50	-525	2550	0
5	37.75	775	250	3325	0

Exercise 5: Hedging a portfolio of stocks

1. Short futures.
2. $N^* = \beta_P \times (V_A / V_F) = 1.75 * (10\,000\,000 / (900 * 200)) \approx 97$ contracts
3.

	Initial Value	Final Value
Portfolio of stocks	10 000 000	11 800 000
Future	900	1017

Gain on the portfolio of stocks = 1 800 000
Loss on the future = (900 – 1017) * 200 * 97 = - 2 269 800

Final Value of the hedged portfolio = $ 9 530 200

4.

	Initial Value	Final Value
Portfolio of stocks	10 000 000	8 200 000
Future	900	783

Loss on the portfolio of stocks = - 1 800 000
Gain on the future = = (900 – 783) * 200 * 97 = 2 269 800

Final Value of the hedged portfolio = $ 10 469 800

5. – Approximate number of future sold (≈ 97 contracts)
   - The correlation between the portfolio of stock and the underlying asset of the future is not perfect.

Exercise 6: Future Arbitrage

1. $35.689
2. Assuming the future quoted price is 37$ (theoretical value of 36.689$), we can conclude that the future is overvalued. The arbitrage strategy to implement is to short the future contract and to buy in the same time an equivalent number of underlying shares. Cash and carry strategy.

Initial Position	
Description	**Cash Flow**
Future sold (100 shares) at $37	$0
100 shares bought at $35.65	- $3565
Loan to finance the shares bought	+ $3565
Net initial Cash Flow	**$0**

Scenario 1: the share is trading at $30.45 – Final position	
Description	**Cash-Flow**
100 shares sold at $30.45 (delivery of 100 shares bought)	+ $3045
Gain/Loss on the short position in the future contract and the delivery of 100 shares at the expiry of the contract ($37 – $30.45) * 100	+ $655
Loan reimbursed (capital + interest at 3% per year for 20 days) (3565 * exp(0.03*20/365)	- $3570
Net Cash Flow at expiry	**+ $130**
Net profit (net initial cash flow + net cashflow at expiry)	**+ $130**

Scenario 2: the share is trading at $39 – Final position	
Description	**Cash-Flow**
100 shares sold at $39 (delivery of 100 shares bought)	+ $3900
Gain/Loss on the short position in the future contract and the delivery of 100 shares at the expiry of the contract ($37 – $39) * 100	- $200
Loan reimbursed (capital + interest at 3% per year for 20 days) (3565 * exp(0.03*20/365)	- $3570
Net Cash Flow at expiry	**+ $130**
Net profit (net initial cash flow + net cashflow at expiry)	**+ $130**

N.B: the dividend is not considered in both scenarios above.

3. Assuming the future quoted price is 34$ (theoretical value of 36.689$), we can conclude that the future is undervalued. The arbitrage strategy to implement is to buy the future contract and to sell in the same time an equivalent number of underlying shares. Reverse cash and carry strategy.

Initial Position	
Description	**Cash Flow**
Future bought (100 shares) at $34	$0
100 shares sold at $35.65	+ $3565
Investment of the money made from the shares sold	- $3565
Net initial Cash Flow	**$0**

Final position - the share is trading at $30.45	
Description	**Cash-Flow**
100 shares bought at $30.45	- $3045
Gain/Loss on the long position in the future contract and the 100 shares bought at the expiry of the contract ($30.45 – $34) * 100	- $355
Money made on the investment (capital + interest at 3% per year for 20 days) (3565 * exp(0.03*20/365)	+ $3570
Net Cash Flow at expiry	**+ $170**
Net profit (net initial cash flow + net cashflow at expiry)	**+ $170**