Solution:

In order to find which team is more consistent, we shall have to compare the coefficient of variation (C.V.) of the two teams.

Team A			Team B		
(X_A)	$x_A = X_A - \bar{X}_A$	x_A^2	(X_B)	$x_B = X_B - \bar{X}_B$	x_B^2
15	+8	64	20	+13	169
10	+3	9	10	+3	9
7	0	0	5	−2	4
5	−2	4	4	−3	9
3	−4	16	2	−5	25
2	−5	25	1	−6	36
$\Sigma X_A = 42$		$\Sigma x_A^2 = 118$	$\Sigma X_B = 42$		$\Sigma x_B^2 = 252$

$$\bar{X}_A = \frac{\Sigma X_A}{N} = \frac{42}{6} = 7 \qquad \bar{X}_B = \frac{\Sigma X_B}{N} = \frac{42}{6} = 7$$

$$\sigma_A = \sqrt{\frac{\Sigma x_A^2}{N}} = \sqrt{\frac{118}{6}} = 4.43 \qquad \sigma_B = \sqrt{\frac{\Sigma x_B^2}{N}} = \sqrt{\frac{252}{6}} = 6.48$$

$$\text{C.V. (Team A)} = \frac{\sigma}{\bar{X}_A} \times 100 \qquad \text{C.V. (Team B)} = \frac{\sigma}{\bar{X}_B} \times 100$$

$$= \frac{4.43}{7} \times 100 = 63.29\% \qquad = \frac{6.48}{7} \times 100 = 92.57\%$$

Ans. C.V. of Team A is less, so Team A is more consistent.

Example 44. Find out which batsman is more consistent in his performance.

	Batsman A	Batsman B
Average Score	46	50
Standard Deviation	25.5	24.43

Solution:

Batsman A	Batsman B
Coefficient of Variation $= \frac{\sigma}{\bar{X}} \times 100$	Coefficient of Variation $= \frac{\sigma}{\bar{X}} \times 100$
$= \frac{25.5}{46} \times 100 = 55.43\%$	$= \frac{24.43}{50} \times 100 = 48.86\%$

Batsman B is more consistent in performance as his coefficient of variation is less than that of Batsman A.

Example 45. Coefficient of variation of two series are 58% and 69%. Their standard deviations are 21.2 and 15.6. What are their means?

Solution:

$$\text{Coefficient of Variation (C.V.)} = \frac{\sigma}{\bar{X}} \times 100$$

$$\bar{X} = \frac{\sigma}{\text{C.V.}} \times 100$$

$$\text{Mean of the first series} = \frac{21.2}{58} \times 100 = 36.55$$

$$\text{Mean of the second series} = \frac{15.6}{69} \times 100 = 22.61$$

Ans. Mean of first series = 36.55; Mean of second series = 22.61

Example 46. Calculate the standard deviation if coefficient of variation is 23.21, number of items is 110 and mean is 21.

Solution:

$$\text{Coefficient of Variation (C.V.)} = \frac{\sigma}{\bar{X}} \times 100$$

$$23.21 = \frac{\sigma}{21} \times 100$$

$$\sigma = \frac{23.21 \times 21}{100} = 4.874$$

Ans. Standard deviation ($\sigma = 4.874$)

Example 47. Particulars regarding the income of two villages are given below:

	Village X	Village Y
Number of People	500	600
Average Income $\bar{X}$	₹ 186	₹ 175
Standard Deviation	9	10

(i) What is the average income of the village X and Y taken together?
(ii) Which village has a larger income?
(iii) In which village, variation in income is greater?

Solution:

(i) Average income of the village X and Y taken together (Combined Mean)

$$\text{Combined Mean } (\bar{X}_{X,Y}) = \frac{N_X\bar{X}_X + N_Y\bar{X}_Y}{N_X + N_Y}$$

$\bar{X}_X = 186$, $\bar{X}_Y = 175$, $N_X = 500$, $N_Y = 600$

$$(\bar{X}_{X,Y}) = \frac{(500 \times 186) + (600 \times 175)}{500 + 600} = \frac{1{,}98{,}000}{1{,}100} = ₹\,180$$

(ii) Income of village X = 500 × 186 = ₹ 93,000
Income of village Y = 600 × 175 = ₹ 1,05,000
Thus, Village Y has a larger income.

(iii) Coefficient of Variation of Village X (C.V.$_X$) $= \frac{\sigma}{\bar{X}_x} \times 100 = \frac{9}{186} \times 100 = 4.84\%$

Coefficient of Variation of Village Y (C.V.$_Y$) $= \frac{\sigma}{\bar{X}_y} \times 100 = \frac{10}{175} \times 100 = 5.71\%$

There is more variability in Village Y.

Ans. (i) Average income of the village X and Y taken together = ₹ 180;
(ii) Village Y has a larger income;
(iii) In village Y, variation in income is greater.

Example 48. For a group of 200 candidates, the mean and standard deviation were found to be 40 and 15. Later on it was discovered that the score 43 was misread as 53. Find the correct mean and standard deviations corresponding to the corrected figure.

Solution:

Calculation of Correct Mean

$$\bar{X} = \frac{\Sigma X}{N}$$

Or, $\Sigma X = \bar{X}N$

$\bar{X} = 40$; $N = 200$

$\Sigma X = 40 \times 200 = 8{,}000$

But 8,000 is a wrong value as one score was misread as 53 instead of 43

Correct ΣX = 8,000 – incorrect item + correct item = 8,000 – 53 + 43 = 7,990

$$\text{Correct } \bar{X} = \frac{\Sigma X}{N} = \frac{7{,}990}{200} = 39.95$$

Calculation of Correct Standard deviation

$$\sigma = \sqrt{\frac{\Sigma X^2}{N} - (\bar{X})^2}$$

$$15 = \sqrt{\frac{\Sigma X^2}{200} - (40)^2}$$

$$15 = \sqrt{\frac{\Sigma X^2}{200} - 1{,}600}$$

Squaring both the sides

$$225 = \frac{\Sigma X^2}{200} - 1{,}600$$

$225 \times 200 = \Sigma X^2 - 1{,}600 \times 200$

$\Sigma X^2 = 3{,}20{,}000 + 45{,}000 = 3{,}65{,}000$

But, it is incorrect value

Correct ΣX^2 = Incorrect ΣX^2 – (Incorrect items)^2 + (Correct items)^2

Correct $\Sigma X^2 = 3{,}65{,}000 - (53)^2 + (43)^2 = 3{,}65{,}000 - 2{,}809 + 1{,}849 = 3{,}64{,}040$

$$\text{Correct } \sigma = \sqrt{\frac{\text{Correct } \Sigma X^2}{N} - (\text{Correct } \bar{X})^2}$$

$$\text{Correct } (\sigma) = \sqrt{\frac{3{,}64{,}040}{200} - (39.95)^2} = \sqrt{1{,}820.2 - 1{,}596} = \sqrt{224.2} = 14.97$$

Ans. Correct Mean = 39.95 marks; Correct Standard Deviation = 14.97 marks

Example 49. Calculate variance and coefficient of variation from the following data:

Values	2	6	10	14
Frequency	4	8	2	1

Solution:

Values (X)	Frequency (f)	fX	$x = X - \bar{X}$	x^2	fx^2
2	4	8	–4	16	64
6	8	48	0	0	0
10	2	20	+4	16	32
14	1	14	+8	64	64
	N = Σf = 15	ΣfX = 90			$\Sigma fx^2 = 160$

$$\text{Arithmetic Mean } (\bar{X}) = \frac{\Sigma fX}{\Sigma f} = \frac{90}{15} = 6$$

$$(\sigma) = \sqrt{\frac{\Sigma x^2}{N}} = \sqrt{\frac{160}{15}} = 3.2659$$

Variance = $\sigma^2 = (3.2659)^2 = 10.66$

$$\text{Coefficient of Variation} = \frac{\sigma}{\bar{X}} \times 100 = \frac{3.2659}{6} \times 100 = 54.43\%$$

Ans. Variance = 10.66; Coefficient of Variation = 54.43%

Example 50. For the following data, calculate: (i) Standard Deviation; (ii) Variance; (iii) Coefficient of Standard Deviation; (iv) Coefficient of Variation.

Class	10–19	20–29	30–39	40–49	50–59
Frequency	4	5	6	2	3

ution:

This is a case of inclusive class-intervals. So, it has to be converted into exclusive series.

Marks (X)	No. of students (f)	Mid-point (m)	$d = m - A$ (A = 24.5)	$d' = \frac{m-A}{C}$ C = 10	fd'	d'^2	fd'^2
9.5–19.5	4	14.5	−10	−1	−4	1	4
19.5–29.5	5	**24.5 (A)**	0	0	0	0	0
29.5–39.5	6	34.5	+10	+1	+6	1	6
39.5–49.5	2	44.5	+20	+2	+4	4	8
49.5–59.5	3	54.5	+30	+3	+9	9	27
	N = Σf = 20				**Σfd′ = 15**		$\Sigma fd'^2 = 45$

(i) Standard deviation (σ) $= \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times C$

$\Sigma fd'^2 = 45$; $N = 20$; $\Sigma fd' = 15$; $C = 10$

$$\sigma = \sqrt{\frac{45}{20} - \left(\frac{15}{20}\right)^2} \times 10 = \sqrt{2.25 - .5625} \times 10 = \sqrt{1.6875} \times 10 = 12.99$$

(ii) Variance $= \sigma^2 = (12.99)^2 = 168.74$

(iii) We know: Coefficient of Standard Deviation $= \frac{\sigma}{\bar{X}}$

$$\text{Mean } (\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 24.5 + \frac{15}{20} \times 10 = 32$$

$$\text{Coefficient of Standard Deviation} = \frac{12.99}{32} = 0.406$$

(iv) Coefficient of Variation (C.V.) $= \frac{\sigma}{\bar{X}} \times 100 = \frac{12.99}{32} \times 100 = 40.6\%$

Ans. (i) Standard Deviation = 12.99; (ii) Variance = 168.74; (iii) Coefficient of Standard Deviation = 0.406; (iv) Coefficient of Variation = 40.6%.

Example 51. From the following data, find out which group is more uniform.

Age Group (years)	No. of Persons	
	Group A	Group B
0–10	5	7
10–20	15	12
20–30	20	22
30–40	25	30
40–50	18	20
50–60	10	5
60–70	7	4

Solution:

In order to find which group is more uniform, we shall have to compare the coefficient of variation (C.V.) of the two groups.

Calculation of Coefficient of Variation (Group A)

Age Group (X)	No. of persons (f)	Mid-points (m)	$d = m - A$ (A = 25)	$d' = \frac{m-A}{C}$ C = 10	fd'	d'^2	fd'^2
0–10	5	5	−20	−2	−10	4	20
10–20	15	15	−10	−1	−15	1	15
20–30	20	**25 (A)**	0	0	0	0	0
30–40	25	35	+10	+1	+25	1	25
40–50	18	45	+20	+2	+36	4	72
50–60	10	55	+30	+3	+30	9	90
60–70	7	65	+40	+4	+28	16	112
	N = Σf = 100				Σfd′ = 94		**$\Sigma fd'^2 = 334$**

To calculate coefficient of variation, we will first calculate standard deviation and arithmetic mean.

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times C = \sqrt{\frac{334}{100} - \left(\frac{94}{100}\right)^2} \times 10$$

$$\sigma = \sqrt{3.34 - 0.883} \times 10 = 15.67$$

$$\text{Mean } (\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 25 + \frac{94}{100} \times 10 = 34.4$$

$$\text{Coefficients of Variation (C.V.)} = \frac{\sigma}{\bar{X}} \times 100 = \frac{15.67}{34.4} \times 100 = 45.55\%$$

Calculation of Coefficient of Variation (Group B)

Age Group (X)	No. of persons (f)	Mid-points (m)	$d = m - A$ (A = 25)	$d' = \frac{m-A}{C}$ C = 10	fd'	d'^2	fd'^2
0–10	7	5	−20	−2	−14	4	28
10–20	12	15	−10	−1	−12	1	12
20–30	22	**25 (A)**	0	0	0	0	0
30–40	30	35	+10	+1	+30	1	30
40–50	20	45	+20	+2	+40	4	80
50–60	5	55	+30	+3	+15	9	45
60–70	4	65	+40	+4	+16	16	64
	N = Σf = 100				Σfd′ = 75		**$\Sigma fd'^2 = 259$**

To calculate coefficient of variation, we will first calculate standard deviation and arithmetic mean.

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times C = \sqrt{\frac{259}{100} - \left(\frac{75}{100}\right)^2} \times 10$$

$$\sigma = \sqrt{2.59 - 0.5625} \times 10 = 14.24$$

$$\text{Mean } (\overline{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 25 + \frac{75}{100} \times 10 = 32.5$$

$$\text{Coefficient of Variation (C.V.)} = \frac{\sigma}{\overline{X}} \times 100 = \frac{14.24}{32.5} \times 100 = 43.82\%$$

Ans. Coefficient of variation of Group B (43.82%) is less than that of Group A (45.55%), so Group B is more uniform.

10.22 PROPERTIES OF STANDARD DEVIATION

1. ***The sum of the square of the deviations of the items from their arithmetic mean is the minimum.*** The sum is less than the sum of the square of the deviations of the items from any other value.

It is made clear with the following illustration:

X	$X-\overline{X}$ $\overline{X}=7$	$(X-\overline{X})^2$	$X-8$	$(X-8)^2$
3	−4	16	−5	25
5	−2	4	−3	9
8	+1	1	0	0
12	+5	25	+4	16
		$\Sigma(X-\overline{X})^2 = 46$		$\Sigma(X-8)^2 = 50$

It is clear from the above example that sum of the squares of deviations from mean (46) is less than the sum of squares of deviations (50) taken from assumed mean.

2. ***Standard deviation is independent of change of origin, i.e.*** value of standard deviation remains the same if in a series, a constant is added (or subtracted) to all observations.

3. ***Standard deviation is affected by change of scale, i.e.*** if all the observations are multiplied or divided by a constant, then the standard deviation also gets multiplied (or divided) by this constant.

4. ***Standard deviation of the combined series:*** Like the arithmetic mean, it is possible to compute combined standard deviations of two or more groups.

(Combined Standard Deviation is discussed in detail in Section 10.23)

5. For a given set of observations, ***standard deviation is never less than mean deviation from mean,*** i.e., Standard Deviation > Mean Deviation from mean.

10.23 COMBINED STANDARD DEVIATION

As we can calculate mean of two or more than two series, we can also compute combined standard deviation of two or more than two series. The formula in case of two series:

$$\sigma_{1,2} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_1d_1^2 + N_2d_2^2}{N_1 + N_2}}$$

Where

$\sigma_{1,2}$ = Combined standard deviation of two groups

σ_1 = Standard deviation of first group

σ_2 = Standard deviation of second group

$d_1^2 = (\overline{X}_1 - \overline{X}_{1,2})^2$

$d_2^2 = (\overline{X}_2 - \overline{X}_{1,2})^2$

$\overline{X}_{1,2}$ = Combined arithmetic mean of two groups

$\overline{X}_1$ = Arithmetic mean of first group

$\overline{X}_2$ = Arithmetic mean of second group

N_1 = Number of observations of first group

N_2 = Number of observations of second group

This formula can be extended upto N number of series. If there are three series, then the combined standard deviation is:

$$\sigma_{1,2,3} = \sqrt{\frac{N_1\sigma_1^2 + N_2\sigma_2^2 + N_3\sigma_3^2 + N_1d_1^2 + N_2d_2^2 + N_3d_3^2}{N_1 + N_2 + N_3}}$$

Where,

$d_1^2 = (\overline{X}_1 - \overline{X}_{1,2,3})^2$, $d_2^2 = (\overline{X}_2 - \overline{X}_{1,2,3})^2$, and $d_3^2 = (\overline{X}_3 - \overline{X}_{1,2,3})^2$

The concept of combined mean will be more clear from the following examples.

Example 52. Find the combined standard deviation from the following data:

	Boys	Girls
Number	30	20
Mean	20	30
Standard deviation	4	5

Solution:

To calculate combined standard deviation, we will have to first calculate combined mean:

$$\text{Combined Mean } (\overline{X}_{1,2}) = \frac{N_1\overline{X}_1 + N_2\overline{X}_2}{N_1 + N_2}$$

$\overline{X}_1 = 20$, $\overline{X}_2 = 30$, $N_1 = 30$, $N_2 = 20$