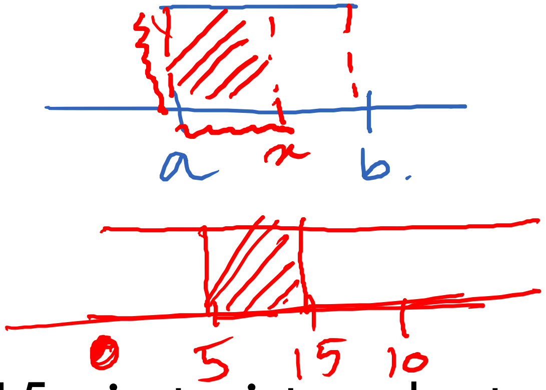
QUANTITATIVE TECHNIQUES FOR MANAGERIAL DECISION - 1 (QTMD1G21-1)

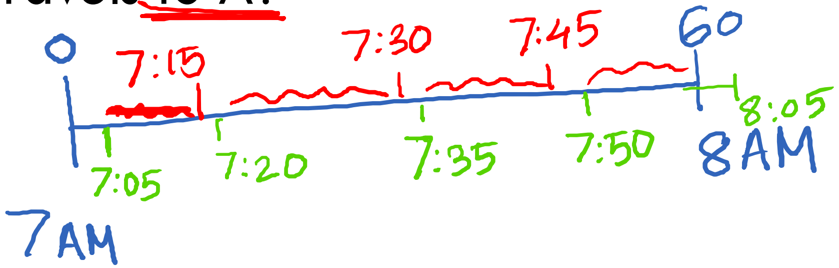
PROBLEM 5.10, TEXTBOOK PG. 212



$f(x) = \begin{cases} \frac{1}{b-a}, & a<x<b \\ 0, & \text{o.w.} \end{cases}$

Trains headed to a destination A arrive at station at 15-minute intervals starting at 7 A.M., whereas trains headed to destination B arrive at 15-minute intervals starting at 7:05 A.M.

a) A certain passenger arrives at the station at a time that is uniformly distributed between 7 and 8 A.M, and then gets on the first train that arrives. What is the probability that the passenger travels to A?



X = Arrival time of passenger.

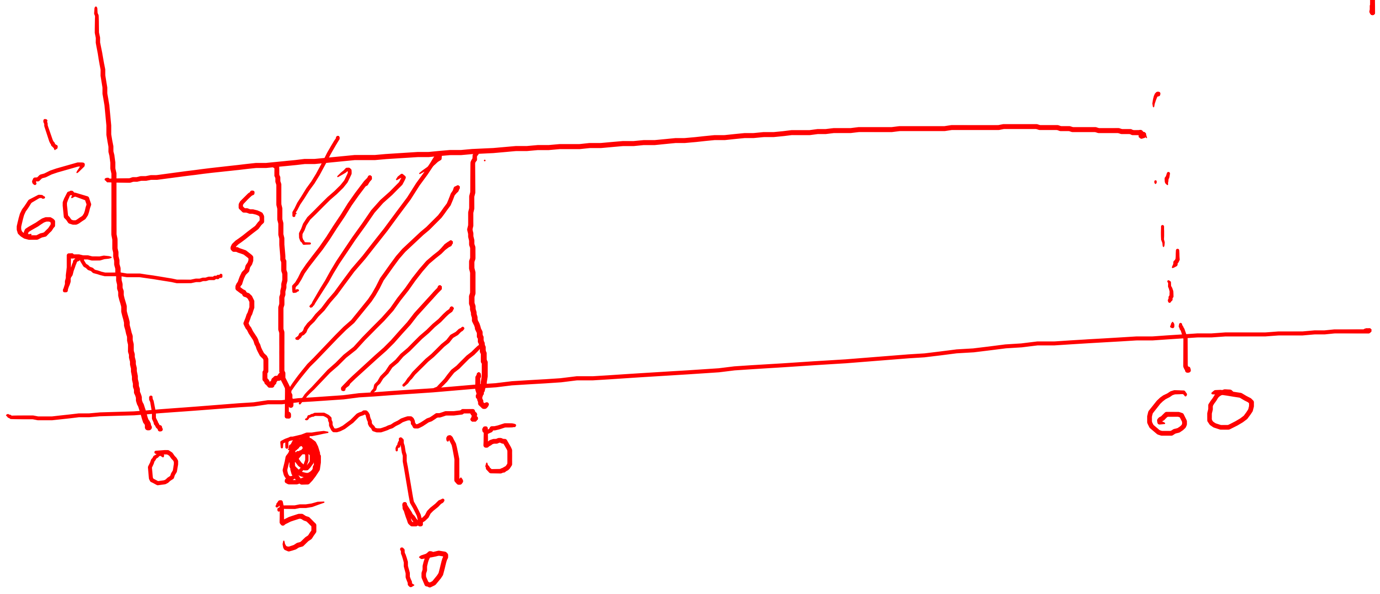
$X \sim U(0, 60)$

$f(x) = \begin{cases} \frac{1}{60}, & 0<x<60 \\ 0, & \text{o.w.} \end{cases}$

P(Travels to A as ~~the first~~ (who gets on the first train which arrives))

$= P(5<X<15) + P(20<X<30) + P(35<X<45) + P(50<X<60)$

$= 10 \times \frac{1}{60} + \dots + 10 \times \frac{1}{60} = \frac{2}{3}$



$$P(5 < X < 15)$$

$$= 10 \times \frac{1}{60}$$

PROBLEM 5.10, TEXTBOOK PG. 212

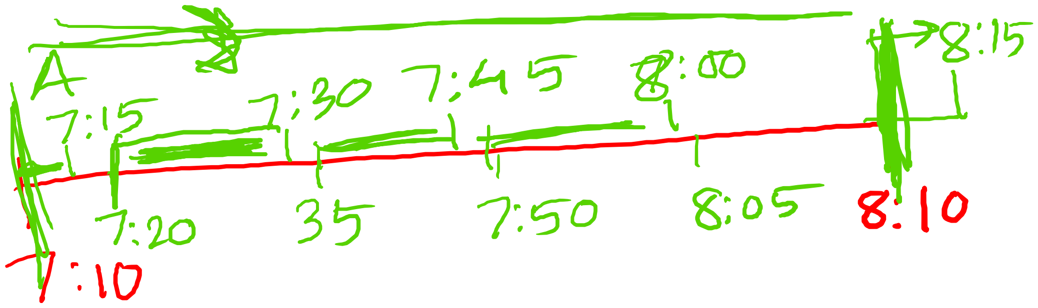
Trains headed to a destination A arrive at station at 15-minute intervals starting at 7 A.M., whereas trains headed to destination B arrive at 15-minute intervals starting at 7:05 A.M.

b) A certain passenger arrives at the station at a time that is uniformly distributed between 7:10 and 8:10 A.M, and then gets on the first train that arrives. What is the probability that the passenger travels to A?

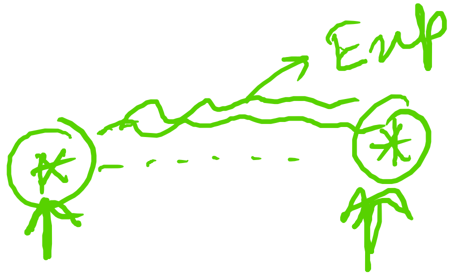
X = Arrival time

$X \sim U(0, 60)$

$P(\text{Travel to A}) = \frac{2}{3}$



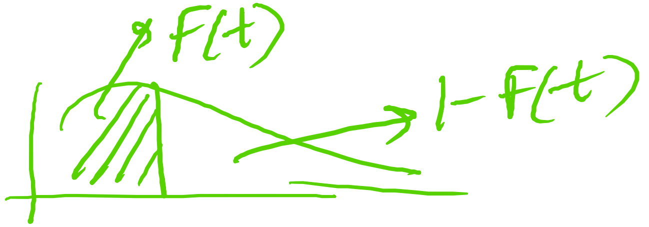
EXPONENTIAL DISTRIBUTION



Used to model the (waiting) time between successive events, e.g., the time between failures of light bulbs, time between two earthquakes etc.

p.d.f./density $f(t) = \begin{cases} \lambda e^{-\lambda t}, \text{for } t > 0 \\ 0, \text{otherwise} \end{cases}$

X ~ Exp(λ), where, λ is the "rate of events".



EXPONENTIAL DISTRIBUTION

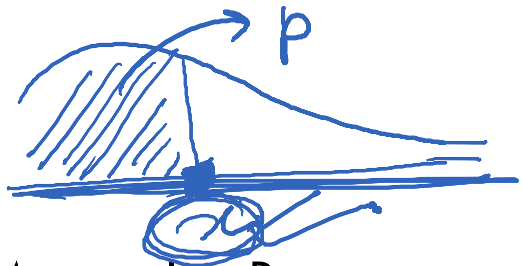
- Mean: $E(X) = \frac{1}{\lambda}$
- Variance: $Var(X) = \frac{1}{\lambda^2}$
- c.d.f. $F(t) = P(X \leq t) = \begin{cases} 1 - e^{-\lambda t}, x \geq 0 \\ 0, \text{otherwise} \end{cases}$
- Survival function: $S(t) = P(X > t) = 1 - F(t) = e^{-\lambda t}$

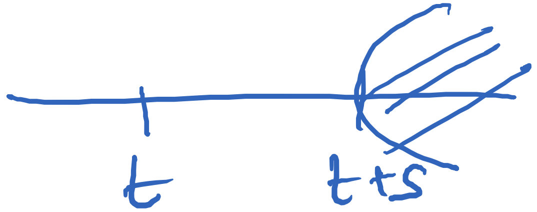
$t > 0$

$$\int_{-\infty}^{t} f(t)\,dt = \int_{0}^{t} \lambda e^{-\lambda t}\,dt$$

$$= -e^{-\lambda t}\Big|_0^t$$

$$F(t) = 1 - e^{-\lambda t}$$





$F(x) = P(X \le x)$

$\Rightarrow 1 - F(x) = P(X > x)$

Now, $F(x) = 1 - e^{-\lambda x}$

EXPONENTIAL DISTRIBUTION

- Memoryless Property:

If X ~ Exp (λ) for some λ > 0, then

$P(X > t + s \mid \cancel{T} > t) = P(X > s)$, where $s \geq 0, t \geq 0$.

which means given survival to time t, the chance of surviving a further time s is the same as the chance of surviving to time s in the first place.

$$P(X > t+s \mid X > t) = \frac{P(X > t+s \cap X > t)}{P(X > t)} = \frac{P(X > t+s)}{P(X > t)} = \frac{1 - F(t+s)}{1 - F(t)}$$

$$= \frac{1 - 1 + e^{-\lambda(t+s)}}{1 - 1 + e^{-\lambda t}} = e^{-\lambda s} = P(X > s)$$

R:

p.d.f.: dexp(t, λ)

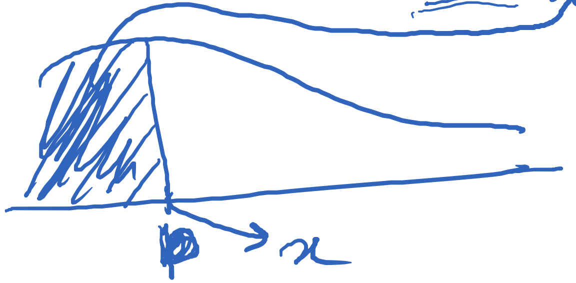
$f(t) = \lambda e^{-\lambda t}$

c.d.f.: pexp(t, λ)

$F(x)$

Quantile: qexp(p, λ)

$F^{-1}(p)$



EXAMPLES:

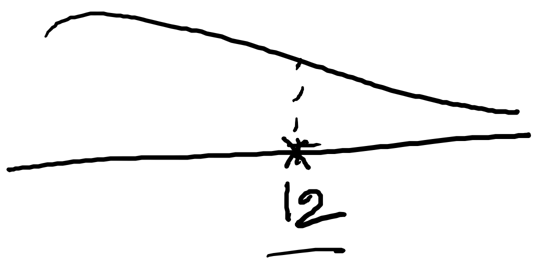
Banks/supermarkets - waiting for service

Computers - waiting for a response

Failure situations - waiting for a failure to occur e.g. in a piece of machinery

Public transport - waiting for a train or a bus

$F(x) = 1 - e^{-\lambda x}$



PROBLEM 5.13, TEXTBOOK PG. 218

The lifetime of a particular mobile phone follows an exponential distribution with mean 30 months. If a person buys a second-hand mobile phone which has been used for some time, what is the probability that it will work for an additional 12 months?

Ans: T = life of mobile phone in months

$T \sim Exp(\lambda) \rightarrow T \sim Exp\left(\frac{1}{30}\right)$

$E(T) = 30$

$\Rightarrow \frac{1}{\lambda} = 30 \Rightarrow \lambda = \frac{1}{30}$

Suppose 1st user used it for t months

$P(T > t+12 \mid T > t) = P(T > 12) = 1 - P(T \leq 12)$

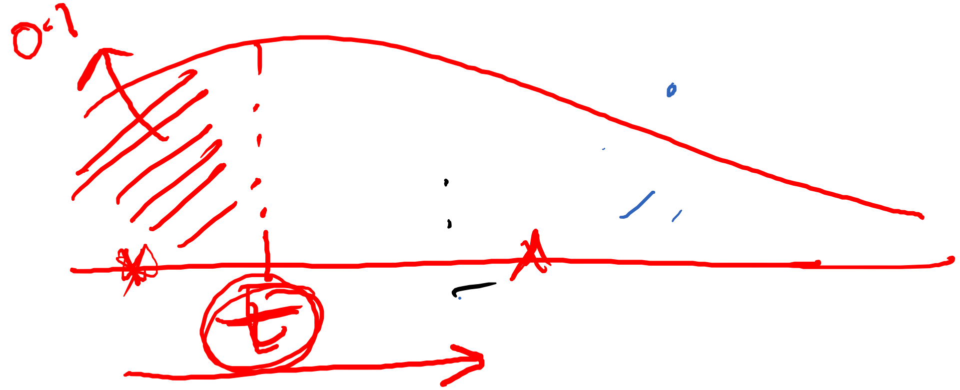
$= 1 - F(12)$

$= 1 - \left(1 - e^{-\frac{1}{30} \times 12}\right)$

$= 0.67032$

In R

$= 1 - pexp(12, 1/30)$



PROBLEM 5.33, TEXTBOOK PG. 214

A toy manufacturing company sells a toy train which has a mean lifetime of 10 months. If the lifetime of the toy train follows an exponential distribution, what should be the guarantee period offered on the train if they do not want to replace more than 10% of the toys?

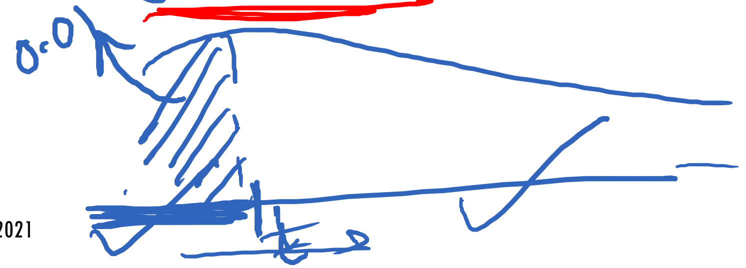
T = life of toy in months

$E(T) = 10 \Rightarrow \lambda = \frac{1}{10}$

$T \sim Exp(\lambda) \Rightarrow T \sim Exp(\frac{1}{10})$

t = guarantee period.

$P(T \leq t) = 0.1$



$\boxed{P(T < t) = 0.1}$

$\Rightarrow 1 - e^{-\lambda t} = 0.1$

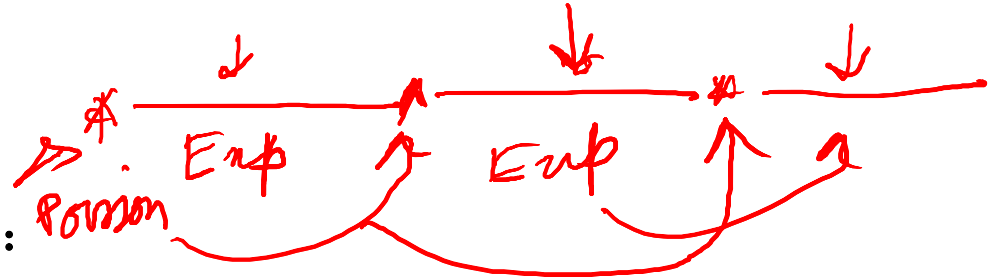
$\Rightarrow t = -10 \times \log_e (0.9)$

$= 1.053605$

In R: qexp(0.1, 1/10)

In R

log(0.9)



ARRIVALS AND TIME BETWEEN ARRIVALS

Counts of Arrival:

- The distribution of arrivals in a fixed interval of a particular length is Poisson
- The number of arrivals in disjoint time intervals are independent.

Times Between Arrivals:

- The distribution of waiting time until the first arrival is exponential,
- The waiting time until the first arrival and the subsequent waiting times between each arrival and the next are independent, all with the same exponential distribution.

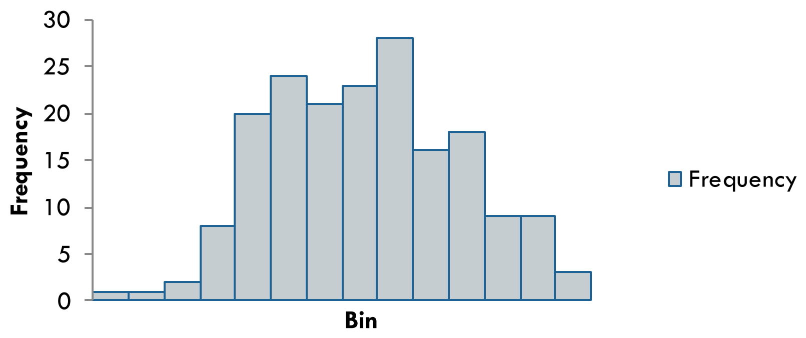
Note:

Expected waiting time between two events is $1/\lambda$.

Number of events in unit time has Poisson(λ) distribution!

NORMAL DISTRIBUTION

Found almost everywhere: considered to be the 'natural distribution' for a number of features for large groups: e.g., Height, Weight, Blood Pressure, Grades ...



Averages for large groups have approximate normal distributions.

→ CLT

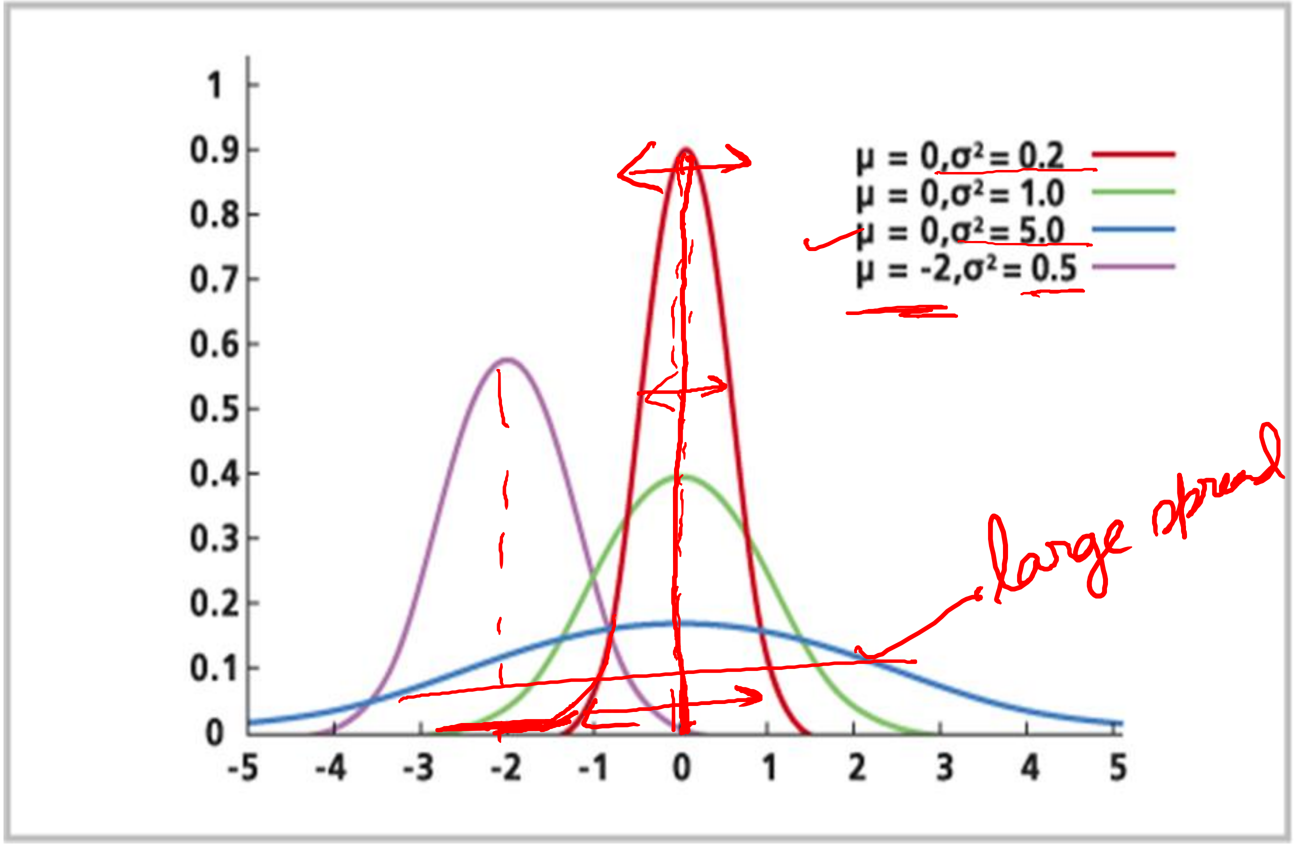
NORMAL DISTRIBUTION

$\to Var(X) = \sigma^2$ $\quad = E(X)$

- The pdf of normal distribution with mean μ and variance σ² is

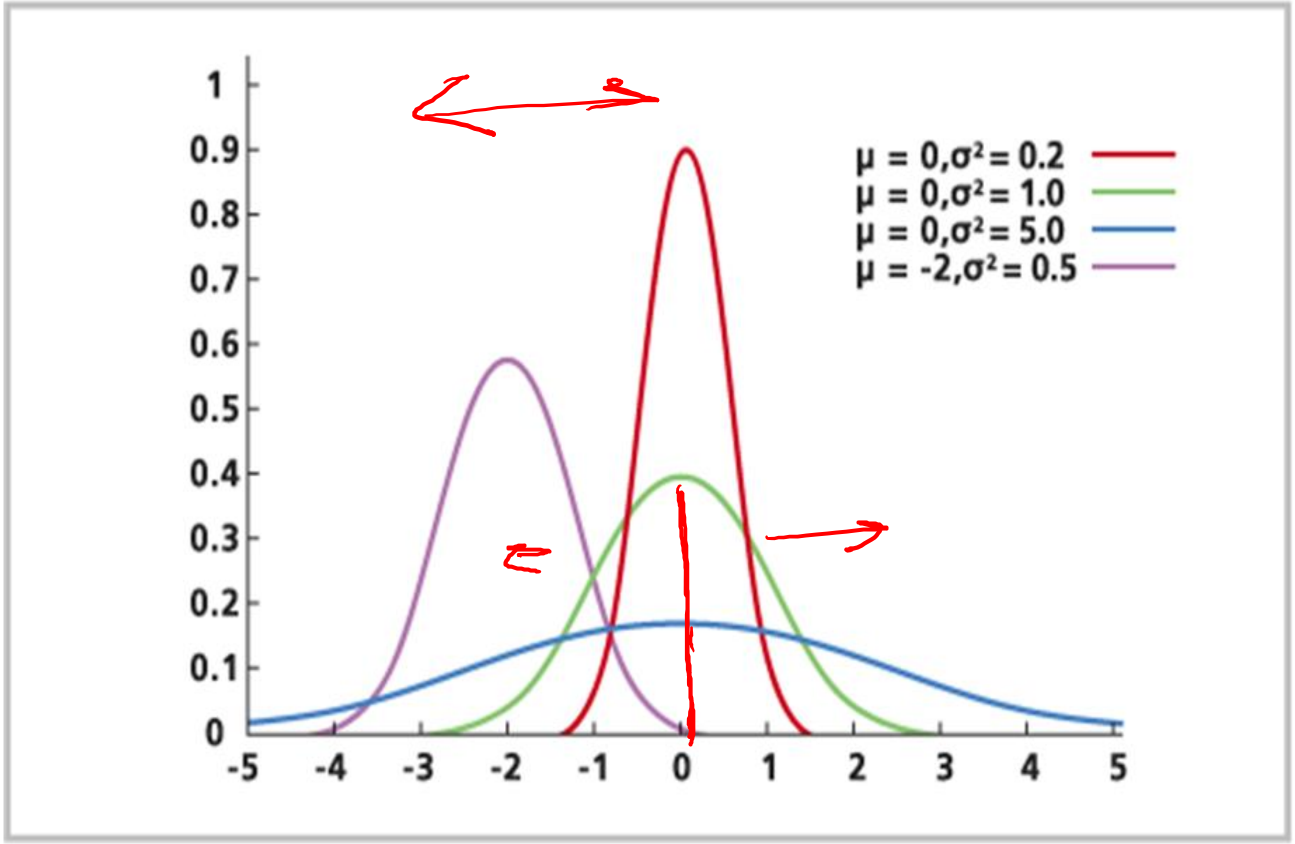
$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{2\sigma^2}}, \text{ for all real } x$$

- If X is normal with mean μ and variance σ², we typically write $X \sim N(\mu,\sigma^2)$.



NORMAL DISTRIBUTION: PROPERTIES

- Normal pdf is symmetric around its mean μ, and its shape depends on sd σ. The higher the sd, the flatter the curve.
- If $X \sim N(\mu, \sigma^2)$, then $aX+b \sim N(a\mu+b, a^2\sigma^2)$.

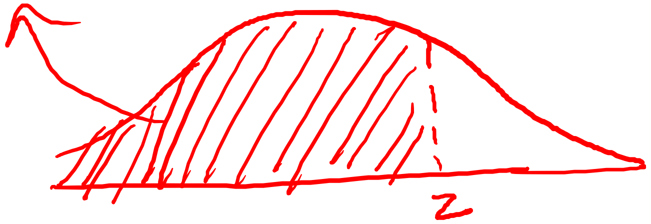


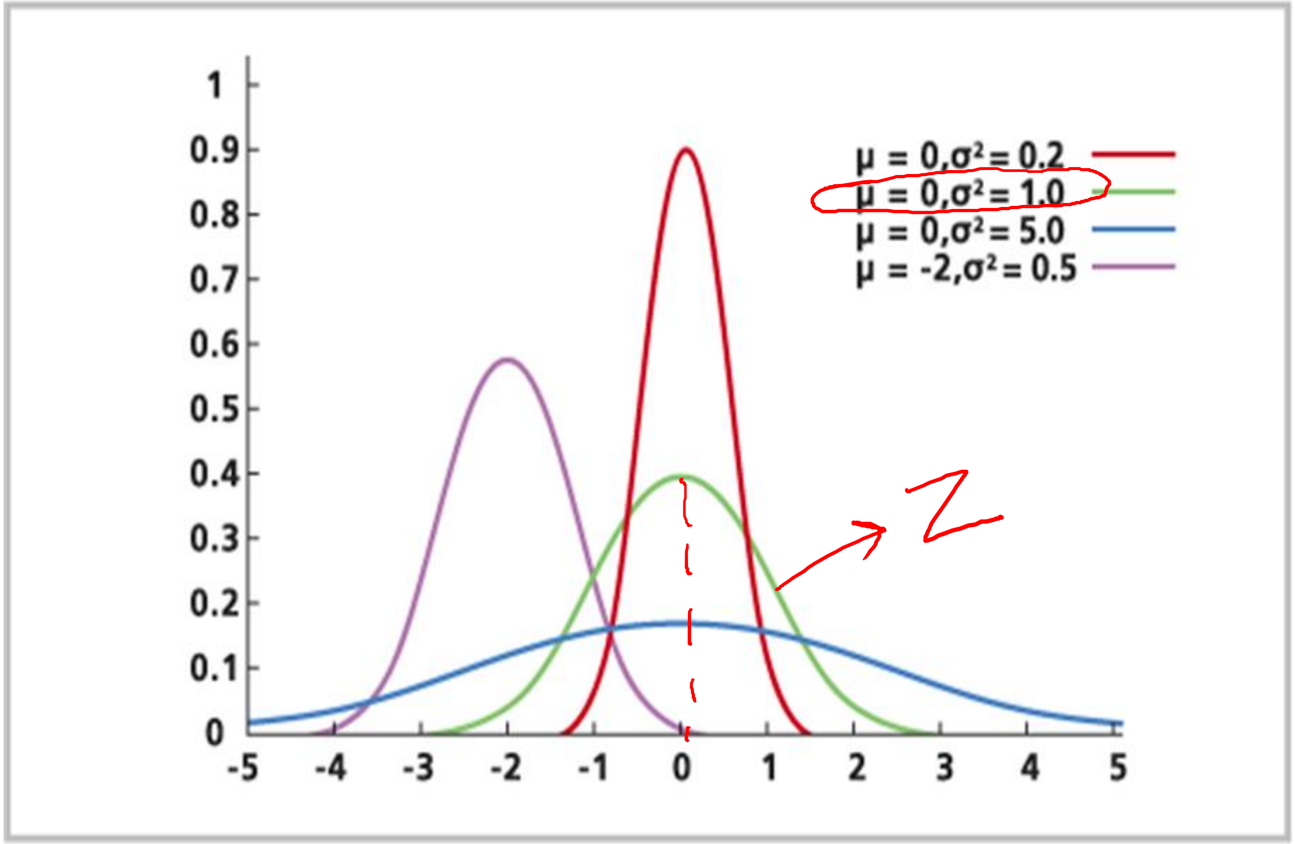
STANDARD NORMAL DISTRIBUTION (Z)

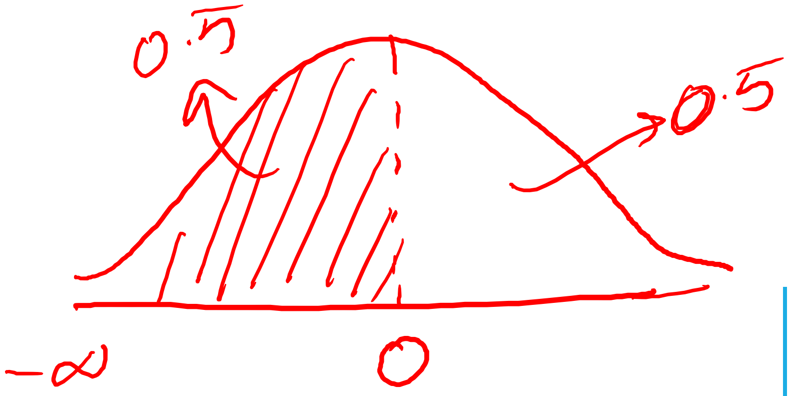
- If $\mu=0$ and $\sigma^2=1$, then X is the standard normal distribution, more commonly denoted by Z.
- We denote its pdf by $\phi(.)$; and cdf by $\Phi(.)$.

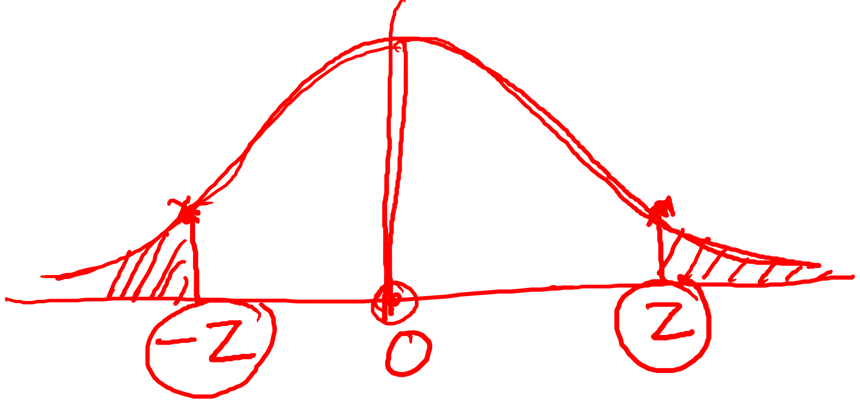
$$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}, \quad -\infty < z < \infty$$

$$\Phi(z) = P(Z \le z)$$









STANDARD NORMAL DISTRIBUTION (Z)

For any normal random variable X, if $X \sim N(\mu,\sigma^2)$, then $Z=(X-\mu)/\sigma \sim N(0,1)$, standard normal.

Notations: pdf: $\phi(.)$; cdf : $\Phi(.)$

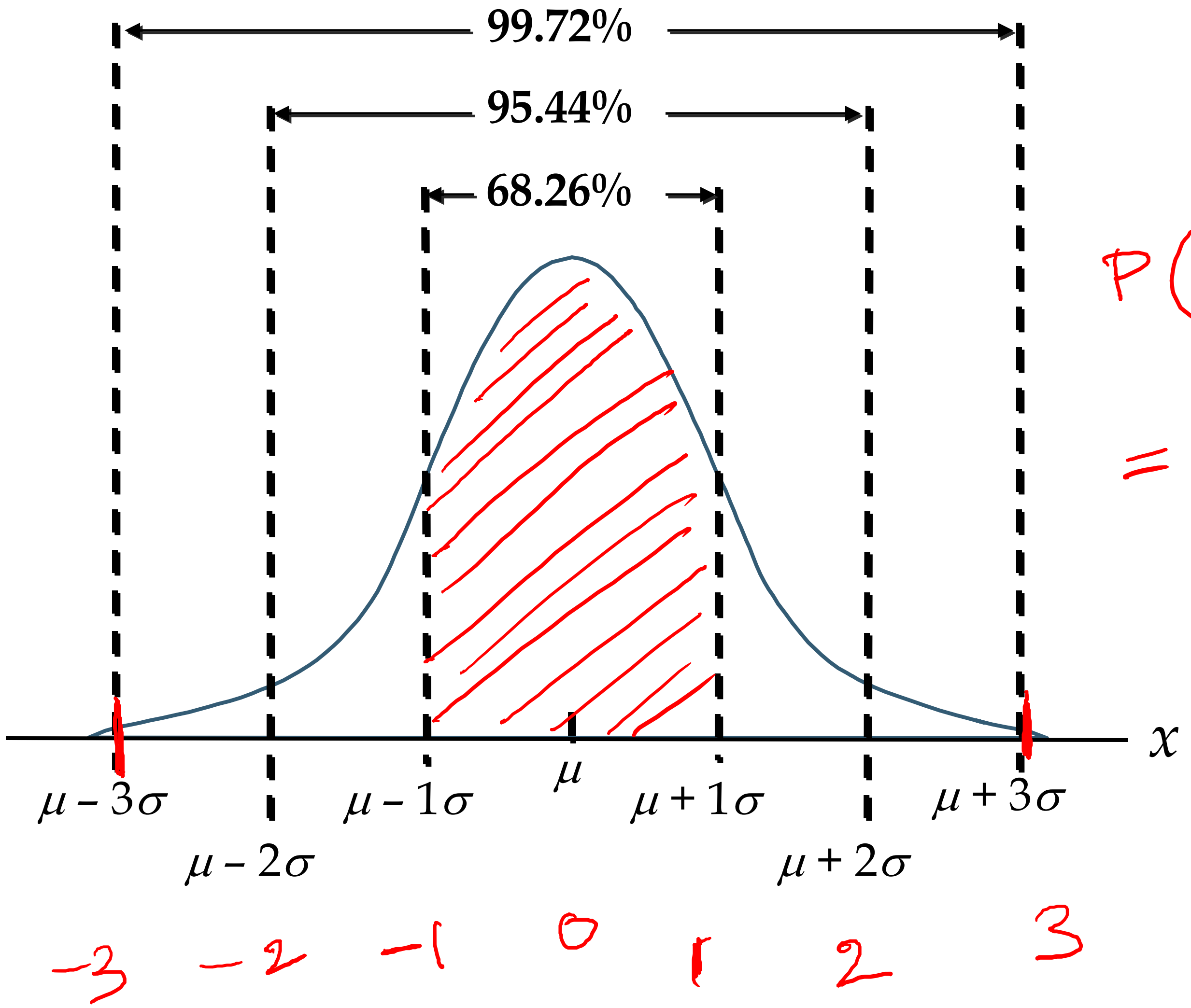
1. The pdf $\phi(.)$ is symmetric around 0, i.e. $\phi(-z) = \phi(z)$.

2. $\Phi(0) = 0.5$ and $\Phi(z) + \Phi(-z) = 1$.

Can re-write: $\Phi(-z) = 1 - \Phi(z)$.

Normal Probability Distribution

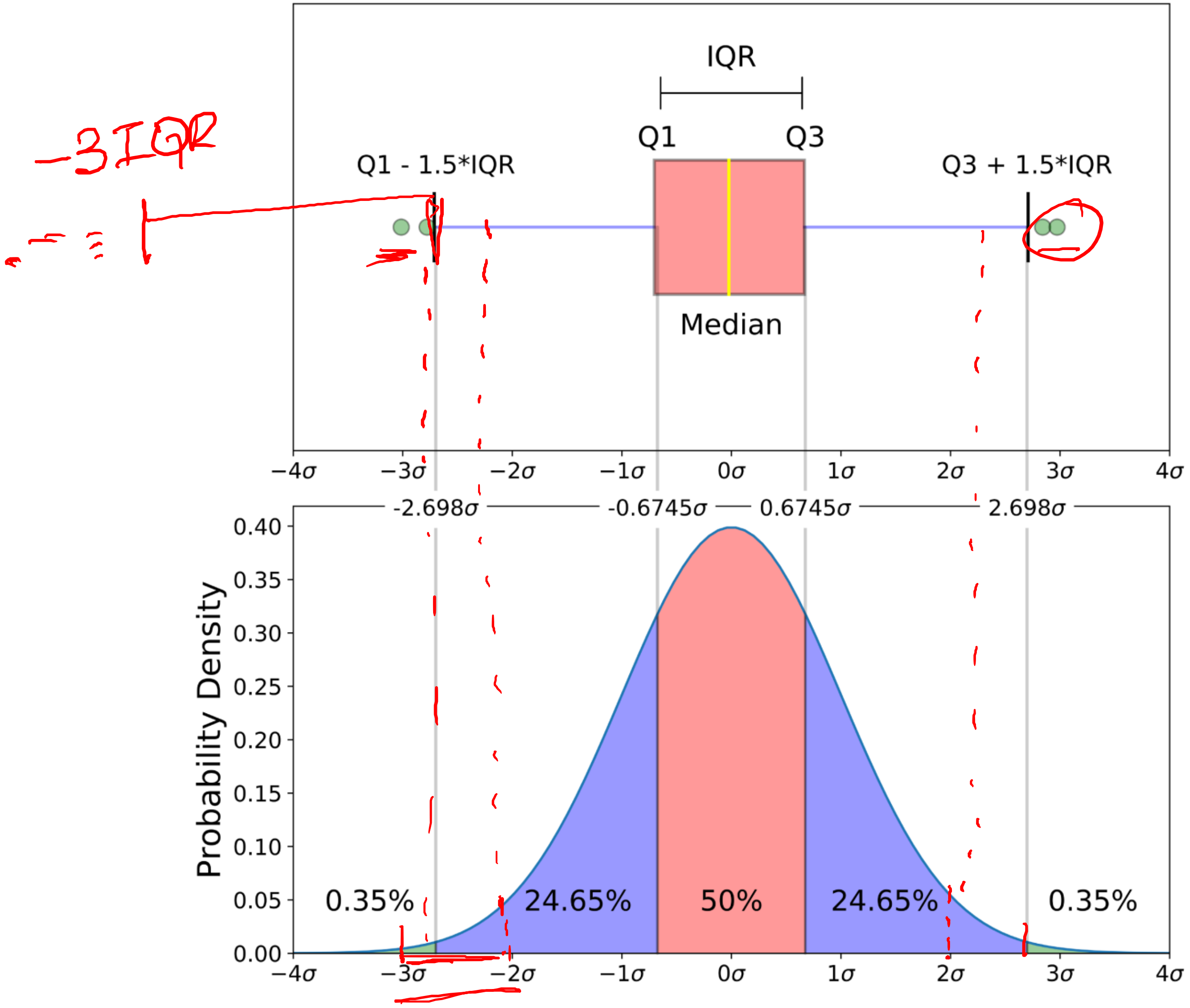
Benchmarking of normal probabilities:



P(-2 < x < 2)

= 0.9544

P(-2 < x < 1.5)



-3IQR
IQR
Q1
Q3
Q1 - 1.5*IQR
Q3 + 1.5*IQR
Median
−4σ −3σ −2σ −1σ 0σ 1σ 2σ 3σ 4σ
-2.698σ
-0.6745σ
0.6745σ
2.698σ
Probability Density
0.40 0.35 0.30 0.25 0.20 0.15 0.10 0.05 0.00
0.35%
24.65%
50%
24.65%
0.35%

EXAMPLE

X~N($\mu = 3, \sigma^2 = 2^2$).

a) What is $P(X \leq 3)$? $= 0.5$

b) What is $P(X \leq 7)$?

c) What is $P(X < 7)$?

$0.5 + \frac{0.9544}{2}$

$= 0.9772$

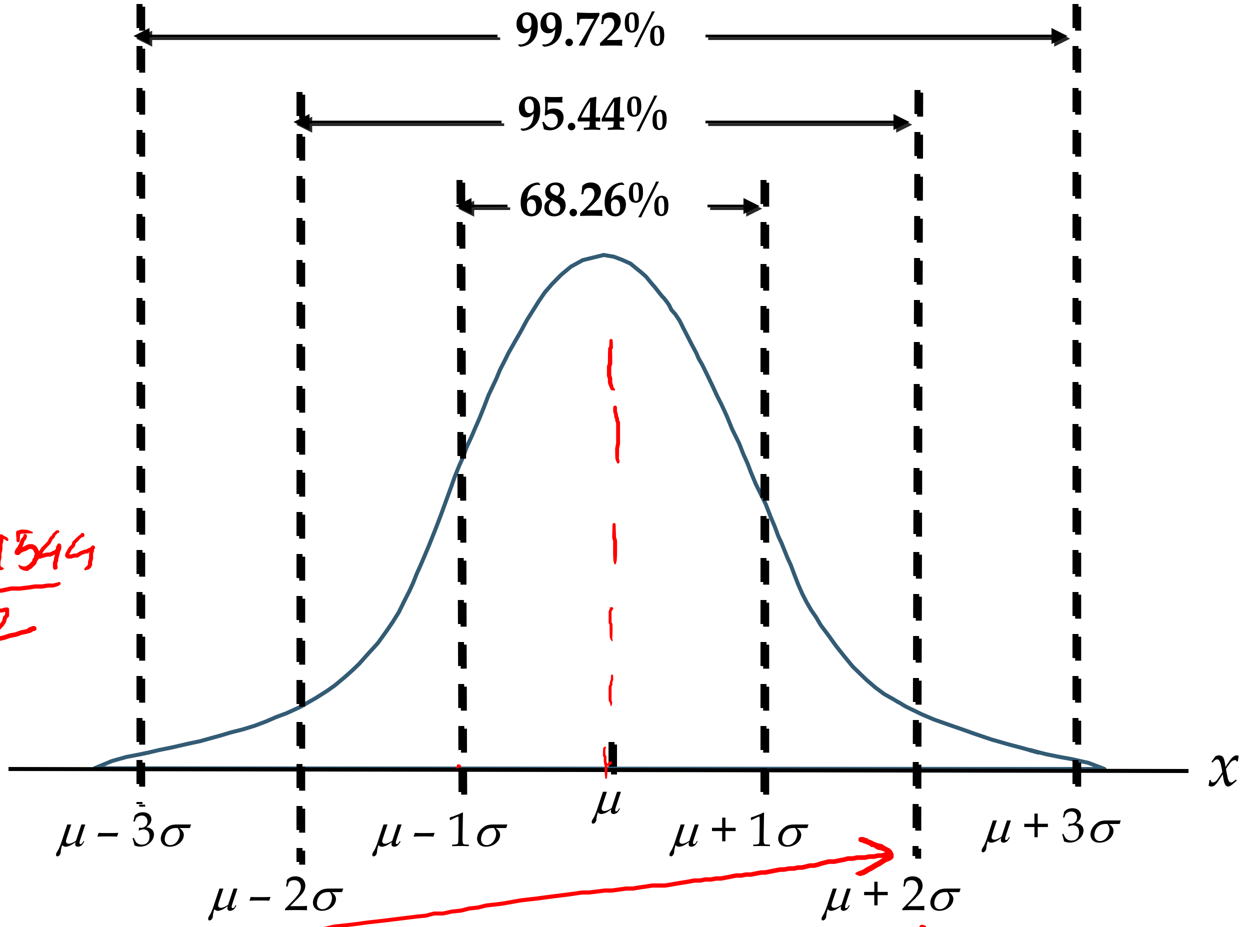
d) What is $P(-3 < X \leq 7)$? $= \frac{0.9972}{2} + \frac{0.9544}{2}$

$= 0.9758$

e) What is $P(-3 < X \leq 2)$?

$P(-3 < X < 2)$

--- Need to use standard normal table now.



-3 $\quad 3-2 = 1$ $\quad 3$ $\quad 3+1 = 4$ $\quad 3+4 = 7$

Standard Normal Probabilities

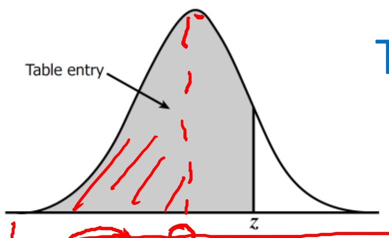


Table 5.1, Page 190, Text

Table entry for z is the area under the standard normal curve to the left of z.

Quantiles

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Probability!

STANDARD NORMAL VALUES $\Phi(Z)$

$\Phi(z)$ is the area under the standard normal pdf $\phi(.)$ up to z.

Notice:

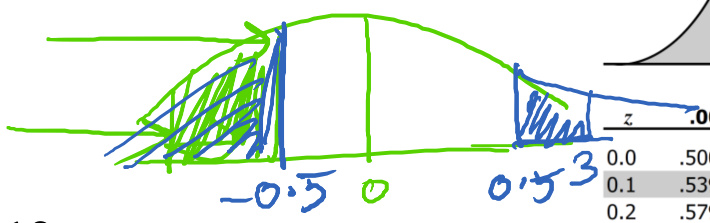
a) $\Phi(0) = 0.5$.

b) Table in book (see left) only lists $\Phi(z)$ for z>0.

Use $\Phi(-z) = 1 - \Phi(z)$ for z<0.

EXAMPLE

$$Z = \frac{X-\mu}{\sigma}$$



$X \sim N(\mu = 3, \sigma^2 = 2^2)$.

e) What is $P(-3 < X \leq 2)$?

Step 1: $P(-3 < X \leq 2)$

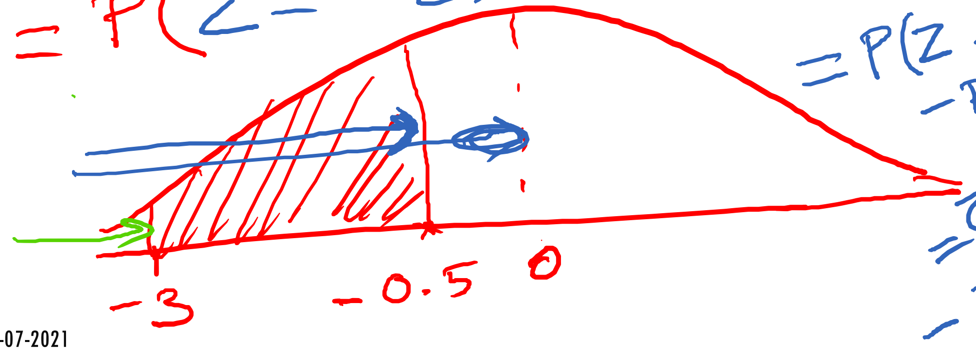
$$= P\left(\frac{-3-3}{2} < \frac{X-3}{2} \leq \frac{2-3}{2}\right)$$

$$= P\left(-3 < Z \leq -\frac{1}{2}\right)$$

$$= P\left(Z \leq -\frac{1}{2}\right) - P(Z \leq -3)$$

$$= P(Z \leq 3) - P\left(Z \leq \frac{1}{2}\right)$$

$$= \Phi(3) - \Phi\left(\frac{1}{2}\right)$$



Standard Normal Probabilities

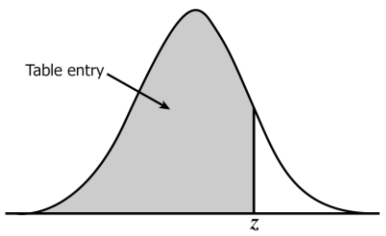


Table entry for z is the area under the standard normal curve to the left of z.

$= 0.9987 - 0.6915 = 0.3072$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

EXAMPLE

$X \sim N(\mu = 3, \sigma^2 = 2^2)$.

f) What is the 95th percentile of X?

Note: 95th percentile/0.95th 'quantile':

x s.t. $F(x) = 0.95 = F^{-1}(0.95)$

Standard Normal Probabilities

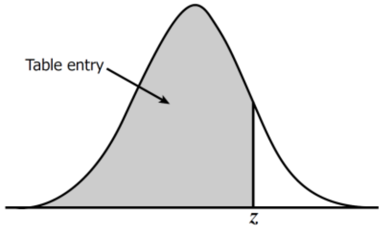


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

R:

`pnorm(x,μ,σ)` ← normal probability (c.d.f.)

`qnorm(x,μ,σ)` ← normal quantile

`pnorm(x)` ← standard normal probability (c.d.f.)

`qnorm(x)` ← standard normal quantile

PROBLEM 5.21, TEXTBOOK PG. 213

Suppose in an apartment complex consisting of 45 families, the total drinking water usage for a day was 1350 lit. If the water usage per family is distributed according to normal distribution with a standard deviation 5 lit,

a) what percentage of families used more than 35 lit of water?

PROBLEM 5.21, TEXTBOOK PG. 213

Suppose in an apartment complex consisting of 45 families, the total drinking water usage for a day was 1350 lit. If the water usage per family is distributed according to normal distribution with a standard deviation 5 lit,

b) what is the probability that exactly 5 families used more than 35 lit of water?

RESULT

For two independent normal random variables $X \sim N(\mu = a, \sigma^2 = v)$ and $Y \sim N(\mu = b, \sigma^2 = u)$,

1. $X+Y \sim N(\mu = a+b, \sigma^2 = v+u)$.
2. $X-Y \sim N(\mu = a-b, \sigma^2 = v+u)$.

Example: $X \sim N(\mu = 5, \sigma^2 = 16)$, $Y \sim N(\mu = -5, \sigma^2 = 9)$, independent.

What is $P(X-Y>15)$?