8

MEASURES OF CENTRAL TENDENCY – ARITHMETIC MEAN

LEARNING OBJECTIVES

8.1 INTRODUCTION

In the previous chapters, we discussed how the raw data can be organized in terms of tables, charts and frequency distributions. We also studied that frequency distributions and graphical representations make raw data more meaningful.

However, sometimes, they fail to convey a clear picture for which it is intended. Therefore, there is a great need for some single measurement, which can describe the main characteristics of the series. Such measures are called *'Measures of Central Tendency'* or *'Average'*.

8.2 MEANING

Measure of Central Tendency is a single value, which is used to represent an entire set of data. It is a typical value to which most of the observations fall closer than to any other value. Measure of Central Tendency is also known as *'Average'* or *'Measure of Location'*.

The following 3 principal measures are widely used in statistical analysis:

(i) Arithmetic Mean;
(ii) Median;
(iii) Mode.

Definitions of Average

- *In the words of Clark and Sekkade,* "An average is an attempt to find one single figure to describe all the figures".
- *In the words of Croxton and Cowden,* "An average is a single value within the range of the data that is used to represent all the values in the series. Since an average is somewhere within the range of data, it is sometimes called a measure of central value".
- *In the words of Spiegal,* "Average is a value which is typical of representative of a set of data".

8.3 OBJECTIVE AND FUNCTIONS OF AVERAGES

The main objectives and functions of averages are:

1. To Present huge data in summarised form: It is very difficult to grasp large number of numerical figures. Averages summarises such data into a single figure, which makes it easier to understand and remember.

   Example: *It is difficult to understand individual families need for water during summers. Therefore, knowledge of the average quantity of water needed for the entire population will help the government in planning water resources.*

2. To make Comparison easier: Averages are very helpful for making comparative studies as they reduce the mass of statistical data to a single figure. Such comparison can be made either at a point of time or over a period of time.

   Example: *Average sales figures of any month can be compared with the preceding months or even with the sales figures of competitive firms for the same months.*

3. To help in Decision-making: Average provides such values, which becomes a guideline for decision makers. Most of the decisions to be taken in research or planning are based on the average value of certain variables.

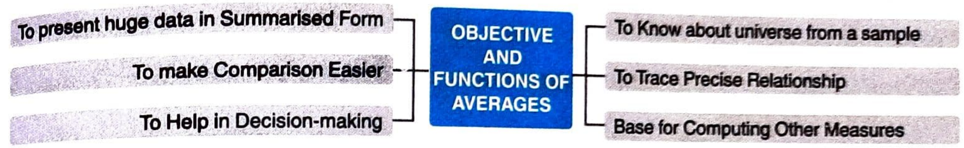
   Example: *If the average monthly sales of a company are falling, the sales manager may have to take certain decisions to improve it.*

4. To know about universe from a sample: Averages also help in obtaining an idea of a complete universe by means of sample data. The average of a sample presents a clear picture of the average of the population.

5. To trace precise relationship: Average becomes essential when it is desired to establish relationships between different groups in quantitative terms.

   Example: *It is vague and irrelevant to say that income of an average American is more than that of an average Indian. It is only a general observation. It is relatively more precise when respective incomes are expressed in terms of averages.*

6. Base for computing other measures: Averages offer a base for computing various other measures like dispersion, skewness, kurtosis that help in many other phases of statistical analysis.



8.4 REQUISITES OF A MEASURE OF CENTRAL TENDENCY

A good measure of average must possess the following characteristics:

1. Rigidly defined: An average should be clear and rigid so that there is no confusion and there is one and only one interpretation.
   - There should not be any chance for applying discretion.
   - Preferably, it should be defined by an algebraic formula, so that the average computed from a set of data by anybody remains the same.
2. Based on all the observations: Average should be calculated by taking into consideration each and every item of the series. If it is not based on all the items, it cannot be said to be representative of the whole group.
3. It should be least affected by fluctuations of sampling: An average should possess sampling stability.
   - If we take two or more independent random samples of the same size from a given population and compute the average for each, then the values so obtained from different samples should not differ much from one another.
   - *For example,* if we select 5 different groups of college students and compute the average age of each group, then average age of the 5 groups should not materially differ from each other.
4. Capable of further Algebraic Treatment: Average should be capable of further mathematical and statistical analysis to expand its utility. *For example,* if separate figures of average marks and number of students of two or more classes are given, then we should be able to compute the combined average.
5. Easy to understand and calculate: The value of an average should be computed by using a simple method without reducing its accuracy and other advantages.

6. Not affected much by Extreme Values: The value of an average should not be affected much by extreme values. If one or two very small or very large items unduly affect the average, the average value may not truly represent characteristics of the entire set of data.

REQUISITES OF A GOOD MEASURE OF CENTRAL TENDENCY

- Rigidly Defined
- Based on all the Observations
- Least Affected by Fluctuations of Sampling
- Capable of further Algebraic Treatment
- Easy to Understand and Calculate
- Not affected much by Extreme Values

8.5 MEASURES OF CENTRAL TENDENCY

The various measures of central tendency or averages commonly used can be broadly classified in the following categories:

TYPES OF AVERAGES

- Mathematical Averages
  - Arithmetic Mean ($\overline{X}$)
  - Geometric Mean (G)
  - Harmonic Mean (H)
- Positional Averages
  - Median (M or Me)
  - Mode (Z)
- Commercial Average
  - Moving Average
  - Progressive Average
  - Composite Average

In accordance with the syllabus prescribed by CBSE for class XI, this book deals only with Arithmetic Mean, Median and Mode (Brief Reference of Quartiles is also given). **In the present chapter, we will discuss** ***'Arithmetic Mean'***. *Median and Mode have been discussed in the Next Chapter.*

8.6 MEANING OF ARITHMETIC MEAN

Arithmetic Mean is defined as the sum of the ***values of all observations divided by the number of observations.***

- It is ordinarily known as 'Mean' or 'Average' by the common man.
- It is usually denoted by $\overline{X}$.

Kinds of Arithmetic Mean

Arithmetic Mean can be computed in two ways:

(i) Simple Arithmetic Mean;

(ii) Weighted Arithmetic Mean.

Let us first understand the calculation of Simple Arithmetic Mean. Its computation can be discussed in the following series: (i) Individual Series; (ii) Discrete Series; and (iii) Continuous Series.

8.7 INDIVIDUAL SERIES

As discussed before, *individual series is the series in which items are listed singly, i.e. each item is given a separate value.* For example, if marks of 10 students in Class XI are given individually, it will form an individual series.

Calculation of Arithmetic Mean in Individual Series

Mean is computed by taking the sum of all observations and dividing the sum by the number of observations in the set. There are 3 methods to calculate arithmetic mean of individual series:

(i) Direct Method;

(ii) Short-Cut Method; and

(iii) Step Deviation Method.

Direct Method

According to this method, all the units are added and then their total is divided by the number of items and the quotient becomes the arithmetic mean.

Steps of Direct Method

1. Let the items (observations) be $X_1, X_2, \ldots\ldots\ldots X_n$.
2. Add up the values of all the items and obtain the total, i.e., ΣX.
3. Find out total number of items in the series, i.e., N.
4. Divide total value of all items (ΣX) by total number of items (N); i.e. $\overline{X} = \frac{\Sigma X}{N}$

{Where, $\overline{X}$ *= Arithmetic Mean;* ΣX *= Sum of all the values of items;* **N** *= Total number of items}*

Example 1. The marks obtained by 10 students in a subject are:

Students	A	B	C	D	E	F	G	H	I	J
Marks	85	60	50	75	55	40	55	70	45	65

Calculate Arithmetic Mean by Direct Method.

Solution:

Calculation of Arithmetic Mean (Direct Method)

Students	Marks (X)
A	85
B	60
C	50
D	75
E	55
F	40
G	55
H	70
I	45
J	65
N = 10	**ΣX = 600**

Total Marks (ΣX) = 600 Marks; Total Number of Students (N) = 10

Arithmetic Mean $(\overline{X}) = \frac{\Sigma X}{N} = \frac{600}{10} = 60$ Marks

Ans. Arithmetic Mean = 60 Marks

Direct Method: Quick Learning

$$\text{Arithmetic Mean } (\overline{X}) = \frac{X_1 + X_2 + \ldots\ldots\ldots + X_n}{N} = \frac{\Sigma X}{N}$$

*The Direct method is generally used when there are few items and the size of the figures is small. If it is not so, there would be considerable difficulty in the calculation of mean. To remove this difficulty, a '**Short-Cut Method**' is used.*

Short-Cut Method

Under this method, any figure is assumed as the mean and deviations are calculated from this assumed mean.

- The need for Short-Cut Method arises when there are large number of observations or it is difficult to compute arithmetic mean by direct method.
- This method is also called '*Assumed Mean Method*'.

Steps of Short-Cut Method

1. Let the items (observations) be $X_1, X_2, \ldots\ldots\ldots X_n$.
2. Decide any item of the series as assumed mean (A)
3. Calculate the deviations (d) of items from assumed mean (A), i.e. deduct A from each item of the series, i.e., X – A
4. Take the sum total of deviations and denote it as Σd
5. Find out the total number of items in the series, i.e., N
6. Apply the following formula: $\overline{X} = A + \frac{\Sigma d}{N}$

{Where, $\overline{X}$ = Arithmetic mean; A = Assumed mean; d = X – A, i.e., deviations of variables from assumed mean; $\Sigma d = \Sigma(X - A)$, i.e., sum of deviations of variables from assumed mean; N = Total number of items}

Example 2. Calculate the arithmetic mean of the marks given in Example 1 by the Short-Cut Method (Assumed Mean Method).

Solution:

Calculation of Arithmetic Mean (Short-cut Method)

Students	Marks (X)	d = X – A (A = 50)
A	85	+ 35
B	60	+ 10
C	50 (A)	0
D	75	+ 25
E	55	+ 5
F	40	– 10
G	55	+ 5
H	70	+ 20
I	45	– 5
J	65	+ 15
N = 10		Σd = 100

In the given example, assumed mean (A) = 50. When deviations (d) from assumed mean is calculated for each student, we get sum total of deviations (Σd) = 100. Given total number of students (N) = 10, the arithmetic mean will be:

Arithmetic Mean $(\overline{X}) = A + \frac{\Sigma d}{N} = 50 + \frac{100}{10} = 60$ Marks

Ans. Arithmetic Mean = 60 Marks

Note:

1. *It should be noted that the answer will remain the same whether Direct Method or Short-cut Method is used.*
2. *In case of individual series, the calculations under short-cut method are more than the direct method. However, in case of discrete series and continuous series, considerable time is saved by adopting the Short-cut Method.*

Step Deviation Method

Step Deviation Method further simplifies the short-cut method. *In this method, deviations from assumed mean are divided by a common factor (C) to get step deviations. Then, these step deviations are used to calculate the value of arithmetic mean.*

Steps of Step Deviation Method

1. Let the items (observations) be $X_1, X_2, \ldots\ldots\ldots X_n$
2. Decide any item of the series as assumed mean (A)
3. Calculate the deviations (d) of items from assumed mean (A), i.e. deduct A from each item of the series
4. Find out common factor (C) from d and calculate d' (step deviations) which is $\frac{d}{C}$
5. Take the sum total of *step deviations* (d') and denote it as Σd'
6. Find out the total number of items in the series, i.e., N
7. Apply the following formula: $\overline{X} = A + \frac{\Sigma d'}{N} \times C$

{Where, $\overline{X}$ = Arithmetic Mean; A = Assumed Mean; d = X – A, i.e., Deviations of variables from Assumed Mean; d' = $\frac{X - A}{C}$, i.e., Step Deviations (deviations divided by common factor); Σd' = Sum of Step Deviations; C = Common Factor; N = Total number of items}

It must be noted that "Step Deviation Method" can be used only when deviations from assumed mean (d) are divisible by a common factor.

Example 3. Calculate the arithmetic mean of the marks given in *Example 1* by the Step Deviation Method.

Solution:

Calculation of Arithmetic Mean (Step Deviation Method)

Students	Marks (X)	d = X − A, A = 50	$d' = \frac{X-A}{C}$, C = 5
A	85	+ 35	+ 7
B	60	+ 10	+ 2
C	**50 (A)**	0	0
D	75	+ 25	+ 5
E	55	+ 5	+ 1
F	40	− 10	− 2
G	55	+ 5	+ 1
H	70	+ 20	+ 4
I	45	− 5	− 1
J	65	+ 15	+ 3
N = 10			**Σd′ = 20**

In the given example, assumed mean (A) = 50. When deviations (d) from assumed mean is calculated and divided by a common factor (C), we get sum total of step deviations (Σd′) = 20. Given total number of students (N) = 10, the arithmetic mean will be:

$$\text{Arithmetic Mean } (\bar{X}) = A + \frac{\Sigma d'}{N} \times C = 50 + \frac{20}{10} \times 5 = 60 \text{ Marks}$$

Ans. 60 Marks

Example 4. Find out the mean height from the following data relating to height measurements of 8 persons in centimeters.

159	161	163	165	167	169	171	173

Solution:

Computation of mean height (Step Deviation Method)

Height (X)	d = X − A, A = 167	$d' = \frac{X-A}{C} = \frac{X-167}{2}$, C = 2
159	− 8	− 4
161	− 6	− 3
163	− 4	− 2
165	− 2	− 1
167 (A)	0	0
169	2	1
171	4	2
173	6	3
N = 8		**Σd′ = − 4**

$$\text{Mean Height } (\bar{X}) = A + \frac{\Sigma d'}{N} \times C = 167 + \frac{-4}{8} \times 2 = 166 \text{ cm}$$

Ans. Mean Height = 166 cm.

Example 5. Following is the marks of 8 students. Find out arithmetic mean by: (i) Direct Method; (ii) Short-Cut Method; (iii) Step Deviation Method.

30	45	60	40	15	65	85	20

Solution:

Computation of Average Marks

Direct Method	Short-Cut Method		Step Deviation Method		
Marks (X)	Marks (X)	d = X − A (A = 40)	Marks (X)	d = X − A (A = 40)	$d' = \frac{X-A}{C}$ (C = 5)
30	30	− 10	30	− 10	− 2
45	45	+ 5	45	+ 5	+ 1
60	60	+ 20	60	+ 20	+ 4
40	40	0	40	0	0
15	15	− 25	15	− 25	− 5
65	65	+ 25	65	+ 25	+ 5
85	85	+ 45	85	+ 45	+ 9
20	20	− 20	20	− 20	− 4
ΣX = 360	**N = 8**	**Σd = 40**	**N = 8**		**Σd′ = 8**
$\bar{X} = \frac{\Sigma X}{N} = \frac{360}{8} = 45$ marks	$\bar{X} = A + \frac{\Sigma d}{N} = 40 + \frac{40}{8} = 45$ marks		$\bar{X} = A + \frac{\Sigma d'}{N} \times C = 40 + \frac{+8}{8} \times 5 = 45$ marks		

Ans. Average Marks = 45 marks

8.8 DISCRETE SERIES

In case of discrete series (ungrouped frequency distribution), values of variable shows the repetitions, i.e., frequencies are given corresponding to different values of variables. In a discrete series, *the total number of observations, i.e., N = Sum total of frequency = Σf.*

Arithmetic mean in a discrete series can be computed by applying:

(i) Direct Method;

(ii) Short-Cut Method; and

(iii) Step Deviation Method.

Direct Method

In the direct method, various items (X) are multiplied with their respective frequencies (f) and the sum of products (ΣfX) is divided by total of frequencies (Σf) to determine simple arithmetic mean, i.e.

$$\bar{X} = \frac{\Sigma fX}{\Sigma f}$$

Steps of Direct Method

1. Multiply different values of variables (X) with respective frequencies (f) and denote it by fX.
2. Obtain the sum total of fX and denote it by ΣfX.
3. Find out the total number of items in the series, i.e., Σf or N
4. Apply the following formula: $\bar{X} = \frac{\Sigma fX}{\Sigma f}$

{Where, $\bar{X}$ = Arithmetic Mean; ΣfX = Sum of the product of variables with the respective frequencies; Σf = Total number of items}

The following example will make the direct method more clear.

Example 6. From the following data of the marks obtained by 60 students of a class, calculate the average marks by the direct method.

Marks	20	30	40	50	60	70
No. of students	8	12	20	10	6	4

Solution:

Calculation of Arithmetic Mean (Direct Method)

Marks (X)	No. of Students (f)	(fX)
20	8	160
30	12	360
40	20	800
50	10	500
60	6	360
70	4	280
	$\Sigma f = 60$	$\Sigma fX = 2{,}460$

Average Marks $(\bar{X}) = \frac{\Sigma fX}{\Sigma f} = \frac{2{,}460}{60} = 41$ marks

Ans. 41 marks

Short-Cut Method

The short-cut method can also be used to calculate the mean in discrete series. This method saves considerable time in calculating mean.

Steps of Short-Cut Method

1. Denote the variable as X and frequency as f.
2. Decide any item of the series as assumed mean (A).
3. Calculate the deviations (d) of items from assumed mean (A), i.e. calculate (X – A) for each item of the series.
4. Multiply the deviations (d) with the respective frequency (f) and obtain the total to get Σfd.
5. Find out the total number of items in the series, i.e., Σf or N.
6. Apply the following formula: $\bar{X} = A + \frac{\Sigma fd}{\Sigma f}$

*{Where, $\bar{X}$ = Arithmetic Mean; **A** = Assumed Mean; **d** = X – A, i.e., deviations of variables from Assumed Mean; Σfd = Sum of the product of deviations (d) with the respective frequencies (f); Σf = Total number of items}*

Let us understand the Short-Cut Method with the help of Example 7:

Example 7. Calculate the arithmetic mean of the marks given in *Example 6* by the Short-cut Method (Assumed Mean Method).

Solution:

Calculation of Average Marks (Short-cut Method)

Marks (X)	No. of Students (f)	d = X – A (A = 40)	fd
20	8	– 20	– 160
30	12	– 10	– 120
40 (A)	20	0	0
50	10	+ 10	+ 100
60	6	+ 20	+ 120
70	4	+ 30	+ 120
	$\Sigma f = 60$		$\Sigma fd = +60$

Average Marks $(\bar{X}) = A + \frac{\Sigma fd}{\Sigma f} = 40 + \frac{+60}{60} = 41$ marks

Ans. 41 Marks

Step Deviation Method

This method is a further simplification of short-cut method. In this method, the values of the deviations (d) are divided by a common factor (C) to ease the calculation process. *The deviations obtained after this division are known as step deviations.*

Steps of Step Deviation Method

1. Denote the variable as X and frequency as f.
2. Decide any item of the series as assumed mean (A).
3. Calculate the deviations (d) of items from assumed mean (A), i e. calculate (X – A) for each item of the series.
4. Find out a common factor (C) from d and calculate d′ (step deviations) which is $\frac{d}{C}$.
5. Multiply step deviations (d′) with frequency (f) and obtain the total to get Σfd′.
6. Find out the total number of items in the series, i.e., Σf or N
7. Apply the following formula: $\overline{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C$

*{Where, $\overline{X}$ = Arithmetic Mean; **A** = Assumed Mean; **C** = Common Factor; **d** = X – A, i.e., deviations of variables from Assumed Mean; **d′** = Step Deviations (deviations from assumed mean divided by common factor); **Σfd′** = Sum of the product of step deviations (d′) with the respective frequencies (f); **Σf** = Total number of items}*

Example 8 will illustrate the computation of arithmetic mean by step deviation method.

Example 8. Calculate the arithmetic mean of the marks given in *Example 6* by the step deviation method.

Solution:

Calculation of Average Marks (Step Deviation Method)

Marks X	No. of Students f	d = X – A (A = 40)	$d' = \frac{X-A}{C}$ (C = 10)	fd′
20	8	– 20	– 2	– 16
30	12	– 10	– 1	– 12
40 (A)	20	0	0	0
50	10	+ 10	+ 1	+ 10
60	6	+ 20	+ 2	+ 12
70	4	+ 30	+ 3	+ 12
	Σf = 60			**Σfd′ = + 6**

Average Marks $(\overline{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 40 + \frac{+6}{60} \times 10 = 41$ marks

Ans. 41 Marks

Example 9. From the following data, calculate arithmetic mean by: (i) Direct Method; (ii) Short-cut Method; (iii) Step Deviation Method.

Variable (X)	2	3	4	5	6	7	8	9
Frequency (f)	10	16	11	8	6	4	3	2

Solution:

Calculation of Arithmetic Mean

Direct Method			Short-Cut Method				Step Deviation Method				
X	f	fX	X	f	d = X – A (A = 5)	fd	X	f	d = X – A (A = 5)	$d' = \frac{X-A}{C}$ (C = 1)	fd′
2	10	20	2	10	– 3	– 30	2	10	– 3	– 3	– 30
3	16	48	3	16	– 2	– 32	3	16	– 2	– 2	– 32
4	11	44	4	11	– 1	– 11	4	11	– 1	– 1	– 11
5	8	40	5	8	0	0	5	8	0	0	0
6	6	36	6	6	+ 1	+ 6	6	6	+ 1	+ 1	+ 6
7	4	28	7	4	+ 2	+ 8	7	4	+ 2	+ 2	+ 8
8	3	24	8	3	+ 3	+ 9	8	3	+ 3	+ 3	+ 9
9	2	18	9	2	+ 4	+ 8	9	2	+ 4	+ 4	+ 8
	Σf = 60	**ΣfX = 258**		**Σf = 60**		**Σfd = – 42**		**Σf = 60**			**Σfd′ = – 42**
$\overline{X} = \frac{\Sigma fX}{\Sigma f} = \frac{258}{60} = 4.30$			$\overline{X} = A + \frac{\Sigma fd}{\Sigma f} = 5 + \frac{-42}{60} = 4.30$				$\overline{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C = 5 + \frac{-42}{60} \times 1 = 4.30$				

Ans. Arithmetic Mean = 4.30

8.9 CONTINUOUS SERIES

In the case of continuous series (grouped frequency distribution), the value of a variable is grouped in various class-intervals (like 10–20, 20–30, etc.) along with their respective frequencies. The process of the calculation of arithmetic average in a continuous series is the same as in case of a discrete series. In a continuous series, **the mid-points of the various class-intervals are used to replace the class-interval. Once it is done,** there is no difference between a continuous series and a discrete series.

Why Mid-Points are important?

The exact value of each frequency is unknown. Therefore, mid-point of each class-interval is assumed to be the true average of the values of all of the frequencies falling within that class-interval. For example, if in a class interval of 0–10, there are 7 frequencies, then it is assumed that all these 7 frequencies have a value of $\frac{0+10}{2} = 5$ each. Hence, it is necessary to determine the mid-points of all the class-intervals.

How to Calculate Mid-Points?

The mid-point are calculated by the formula: Mid-point = $\frac{l_1 + l_2}{2}$

Where: l_1 = Lower limit of Class-interval; l_2 = Upper limit of Class-interval.

For Example, The mid-point of class-interval 10–20 will be: $\frac{10+20}{2} = 15$.

In the continuous series also, the following three methods are used to calculate arithmetic mean:

(i) Direct Method;

(ii) Short-Cut Method; and

(iii) Step Deviation Method.

Direct Method

The direct method under continuous series is the same as under discrete series, except that we first convert the continuous series into discrete series by taking the mid-points of each class-interval.

Steps of Direct Method

1. Calculate the mid-point of each class-interval and denote it by m.
2. Multiply the mid-points (m) with respective frequencies (f) and denote it by fm.
3. Obtain the sum total of fm and denote it by Σfm.
4. Find out the total number of items in the series, i.e., Σf or N.
5. Apply the following formula: $\overline{X} = \frac{\Sigma fm}{\Sigma f}$.

*{Where, $\overline{X}$ = Arithmetic mean; **Σfm** = Sum of the product of mid-points with the respective frequencies; **Σf** = Total number of items}*

The Direct Method will be more clear with the help of Example 10:

Example 10. The following table gives the marks in English secured by 30 students of a class in their weekly test:

Marks	0–5	5–10	10–15	15–20	20–25
No. of Students	2	8	6	10	4

Calculate the average marks of students by the direct method.

Solution:

Calculation of Average Marks (Direct Method)

Marks (X)	No. of Students (f)	Mid-value (m)	fm
0–5	2	2.5	5
5–10	8	7.5	60
10–15	6	12.5	75
15–20	10	17.5	175
20–25	4	22.5	90
	Σf = 30		Σfm = 405

Average Marks $(\overline{X}) = \frac{\Sigma fm}{\Sigma f} = \frac{405}{30} = 13.50$ marks

Ans. Average marks = 13.50

Short-Cut Method

The short-cut method in case of continuous series saves considerable time in calculating mean.

Steps of Short-Cut Method

1. Calculate the mid-point of each class-interval and denote it by m.
2. Decide any one mid-point as the assumed mean (A).
3. Calculate the deviations (d) of mid-points from the assumed mean (A), i.e. calculate (m – A).
4. Multiply the deviations (d) with the respective frequency (f) and obtain the total to get Σfd.
5. Find out the total number of items in the series, i.e., Σf or N.
6. Apply the following formula: $\overline{X} = A + \frac{\Sigma fd}{\Sigma f}$.

*{Where, $\overline{X}$ = Arithmetic Mean; **A** = Assumed Mean; **d** = m – A, i.e., deviations of mid-points from assumed mean; **Σfd** = Sum of the product of deviations (d) with the respective frequencies (f); **Σf** = Total number of items}*

The use of Short-Cut Method will be clear from the Example 11:

Example 11. Calculate the arithmetic mean of the marks given in Example 10 by the short-cut method.

Solution:

Calculation of Average Marks (Short-Cut Method)

Marks X	No. of Students f	Mid-value m	d = m – A (A = 12.5)	fd
0–5	2	2.5	– 10	– 20
5–10	8	7.5	– 5	– 40
10–15	6	12.5 (A)	0	0
15–20	10	17.5	+ 5	+ 50
20–25	4	22.5	+ 10	+ 40
	Σf = 30			Σfd = + 30

Average Marks $(\overline{X}) = A + \frac{\Sigma fd}{\Sigma f} = 12.5 + \frac{+30}{30} = 13.50$ marks

Ans. Average Marks = 13.50

Step Deviation Method

When class-intervals for all the classes in a continuous series are of same magnitude (width), then short-cut method can be further simplified by the step-deviation method.

Steps of Step Deviation Method

1. Calculate the mid-point of each class-interval and denote it by m.
2. Decide any one mid-point as the assumed mean (A).
3. Calculate the deviations (d) of mid-points from the assumed mean (A), i.e. calculate (m – A).
4. Find out a common factor (C) from d and calculate d′ (step deviations) which is $\frac{d}{C}$.
5. Multiply step deviations (d′) with frequency (f) and obtain the total to get Σfd′.
6. Find out the total number of items in the series, i.e., Σf or N.
7. Apply the following formula: $\overline{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C$.

*(Where, $\overline{X}$ = Arithmetic Mean; **A** = Assumed mean. **C** = Common Factor; **d** = m – A, i.e., deviations of mid-points (m) from assumed mean; **d′** = Step Deviations (deviations from assumed mean divided by common factor); **Σfd′** = Sum of the product of step deviations (d′) with respective frequencies (f); **Σf** = Total number of items)*

The step deviation method will be more clear by Example 12:

Example 12. Calculate the arithmetic mean of the marks given in *Example 10* by the step deviation method.

Solution:

Calculation of Average Marks (Step Deviation Method)

Marks X	No. of Students f	Mid-value m	d = m – A (A = 12.5)	$d' = \frac{m-A}{C}$ (C = 5)	fd′
0–5	2	2.5	– 10	– 2	– 4
5–10	8	7.5	– 5	– 1	– 8
10–15	6	**12.5 (A)**	0	0	0
15–20	10	17.5	+ 5	+ 1	+ 10
20–25	4	22.5	+ 10	+ 2	+ 8
	Σf = 30				**Σfd′ = 6**

Average Marks $(\overline{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 12.5 + \frac{+6}{30} \times 5 = 13.50$ marks

Ans. Average marks = 13.50

Explore More

For calculating arithmetic mean in a continuous series, the following assumptions are made:

1. The class-intervals must be closed.
2. The width of each class-interval should be equal.
3. The values of the observations in each class-interval must be uniformly distributed between its lower and upper limits.
4. The mid-value of each class-interval must represent the average of all values in that class.

Example 13. The following table shows the marks obtained by 90 students in a certain examination. Calculate the average marks by: (i) Direct Method; (ii) Short-cut Method; (iii) Step Deviation Method.

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of Students	3	8	12	16	19	16	11	5

Solution:

Calculation of Average Marks

m = mid-point; f = No. of Students

	Direct Method			Short-cut Method		Set deviation Method		
Marks	f	m	fm	d = m – A (A = 45)	fd	d = m – A (A = 45)	$d' = \frac{m-A}{C}$ (C = 10)	fd′
0–10	3	5	15	– 40	– 120	– 40	– 4	– 12
10–20	8	15	120	– 30	– 240	– 30	– 3	– 24
20–30	12	25	300	– 20	– 240	– 20	– 2	– 24
30–40	16	35	560	– 10	– 160	– 10	– 1	– 16
40–50	19	45	855	0	0	0	0	0
50–60	16	55	880	+ 10	+ 160	+ 10	+ 1	+ 16
60–70	11	65	715	+ 20	+ 220	+ 20	+ 2	+ 22
70–80	5	75	375	+ 30	+ 150	+ 30	+ 3	+ 15
	Σf = 90		**Σfm = 3,820**		**Σfd = –230**			**Σfd′ = –23**

$\overline{X} = \frac{\Sigma fm}{\Sigma f} = \frac{3,820}{90} = 42.44$

$\overline{X} = A + \frac{\Sigma fd}{\Sigma f} = 45 + \frac{-230}{90} = 42.44$

$\overline{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C = 45 + \frac{-23}{90} \times 10 = 42.44$

Ans. Average Marks = 42.44

8.10 CHARLIER'S ACCURACY CHECK

Some errors may be made while calculating the value of mean. When the arithmetic mean of a frequency distribution is calculated by shortcut or step-deviation method, the accuracy of the calculations can be checked by using the following formulae given by Charlier.

$\Sigma f(d + 1) = \Sigma fd + \Sigma f$ (for short-cut Method)

$\Sigma f(d' + 1) = \Sigma fd' + \Sigma f$ (for step deviation Method)

If the two sides of these equations do not tally, the calculations are incorrect.

The Charlier's Accuracy Check will be more clear from the following example:

Inclusive Class-Intervals

When the data is given in inclusive series, then it is not necessary to adjust the classes for calculating arithmetic mean as the mid-value remains the same whether the adjustment is made or not. So, **inclusive class-intervals are not converted into an exclusive class-interval series.**

However, in case of median and mode (discussed in the next chapter), the inclusive series have to be converted into exclusive series.

Example 25. Find mean of the following data:

Class-Interval	50–59	40–49	30–39	20–29	10–19	0–9
Frequency	1	3	8	10	15	3

Solution:

In the given example, it is neither necessary to convert the data into exclusive class-interval series nor to arrange the data in ascending order.

Computation of Mean

Class-Interval (X)	Frequency (f)	Mid-value (m)	$d = m - A$ (A = 24.5)	$d' = \frac{m-A}{C}$ (C = 10)	fd'
50–59	1	54.5	+ 30	+ 3	3
40–49	3	44.5	+ 20	+ 2	6
30–39	8	34.5	+ 10	+ 1	8
20–29	10	**24.5 (A)**	0	0	0
10–19	15	14.5	– 10	– 1	– 15
0–9	3	4.5	– 20	– 2	– 6
	$\Sigma f = 40$				$\Sigma fd' = -4$

$$\bar{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C = 24.5 + \frac{-4}{40} \times 10 = 23.50$$

Ans. Mean = 23.50

Open-end Series

Open-end class-intervals are those which do not have the lower limit of the first class-interval and the upper limit of the last class-interval. For example, 'less than 10', or 'more than 100' are open end class-interval.

Calculation of Mean in Open-end Class-Intervals

In such cases, mean cannot be found out unless we assume the missing class limits. The missing values depend on the pattern of class-intervals of other classes.

If the given class-intervals are not equal, then it poses some difficulty in deciding the limits of the open-end classes. In such cases, limits have to be assumed on some rational basis.

The calculation of mean under open-end distribution with equal class-intervals and unequal class-intervals will be clear with the help of Examples 26 and 27 respectively.

Example 26. The following table gives the distribution of daily income of 60 families. Calculate the arithmetic mean.

Daily income (₹)	Below 75	75–150	150–225	225–300	300–375	375–450	450 and over
No. of families	6	17	20	6	5	4	2

Solution:

In the given example, the class-intervals are uniform, i.e., 75. So, we can assume that class-intervals of open-end classes are also equal to 75. It means, the lower limit of the first class-interval is zero (i.e. 0–75) and the upper limit of the last class is 525 (i.e. 450–525).

Now, the arithmetic mean can be calculated by arranging the frequency distribution.

Computation of Average Daily Income

Daily Income (₹) (X)	No. of Families (f)	Mid-value (m)	$d = m - A$ (A = 262.5)	$d' = \frac{m-A}{C}$ (C = 75)	fd'
0–75	6	37.5	– 225	– 3	– 18
75–150	17	112.5	– 150	– 2	– 34
150–225	20	187.5	– 75	– 1	– 20
225–300	6	**262.5 (A)**	0	0	0
300–375	5	337.5	+ 75	+ 1	+ 5
375–450	4	412.5	+ 150	+ 2	+ 8
450–525	2	487.5	+ 225	+ 3	+ 6
	$\Sigma f = 60$				$\Sigma fd' = -53$

$$\text{Average Daily Income } (\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 262.5 + \frac{-53}{60} \times 75 = ₹\,196.25$$

Ans. Average Daily Income = ₹ 196.25

Example 27. Calculate mean of the following series:

Marks	Below 20	20–50	50–90	90–140	Above 140
No. of Students	10	20	40	15	15

Solution:

In the given example, the width of the second class-interval is 30 and of the third class-interval is 40. The width of the fourth interval is 50. It means that the width is rising by 10. On this assumption, the magnitude of the first class shall be 20 and of the last class 60. The class-intervals would then be:

Marks (X)	No. of Students (f)	Mid-value (m)	fm
0–20	10	10	100
20–50	20	35	700
50–90	40	70	2,800
90–140	15	115	1,725
140–200	15	170	2,550
	$\Sigma f = 100$		$\Sigma fm = 7,875$

Average Marks $(\bar{X}) = \frac{\Sigma fm}{\Sigma f} = \frac{7,875}{100} = 78.75$ marks

Ans. Average Marks = 78.75 Marks

Unequal Class-Intervals

Sometimes the class-interval of the distribution is unequal. In such cases, mean can be determined in the usual manner after calculating the mid-values of each interval. *It means, class-intervals are not made equal.*

This will be clear with the help of Example 28:

Example 28. Calculate arithmetic mean from the following data:

Marks	0–10	10–20	20–40	40–70	70–100
No. of Students	8	12	30	6	4

Solution:

In the given example, the class-intervals are unequal. Mean will be calculated directly after calculating the mid-points.

Marks	Mid-value (m)	No. of Students (f)	fm
0–10	5	8	40
10–20	15	12	180
20–40	30	30	900
40–70	55	6	330
70–100	85	4	340
		$\Sigma f = 60$	$\Sigma fm = 1,790$

Average Marks $(\bar{X}) = \frac{\Sigma fm}{\Sigma f} = \frac{1,790}{60} = 29.83$ Marks

Ans. Average marks = 29.83 Marks

SUMMARY OF MEAN IN SPECIAL CASES

CASE 1a: Cumulative Frequency Distribution *(Less Than Series)*: Convert it into Simple Frequency Distribution and then calculate Mean in usual manner.

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50
No. of Stud.	9	16	24	28	35

Marks (X)	No. of Stud. (f)	Mid value (m)	d = m – A, A = 25	$d' = \frac{m-A}{C}$, C = 10	fd′
0 – 10	9	5	–20	–2	–18
10 – 20	7	15	–10	–1	–7
20 – 30	8	25	0	0	0
30 – 40	4	35	10	1	4
40 – 50	7	45	20	2	14
	$\Sigma f = 35$				$\Sigma fd' = -7$

Mean $(\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 25 + \frac{-7}{35} \times 10 = 23$ Marks

CASE 1b: Cumulative Frequency Distribution *(More Than Series)*: Convert it into Simple Frequency Distribution and then calculate Mean in usual manner.

Marks	More than 10	More than 20	More than 30	More than 40	More than 50
No. of Stud.	30	22	18	12	5

Marks (X)	No. of Stud. (f)	Mid value (m)	d = m – A, A = 35	$d' = \frac{m-A}{C}$, C = 10	fd′
10 – 20	8	15	–20	–2	–16
20 – 30	4	25	–10	–1	–4
30 – 40	6	35	0	0	0
40 – 50	7	45	10	1	7
50 – 60	5	55	20	2	10
	$\Sigma f = 30$				$\Sigma fd' = -3$

Mean $(\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 35 + \frac{-3}{30} \times 10 = 34$ Marks

CASE 2: Mid-Values are Given: There is no need to convert such mid-values into regular Class-Intervals, i.e. mean is calculated with the help of mid-points only.

Mid-Points	5	15	25	35	45
Frequency	10	20	30	20	10

Mid-Points (m)	Frequency (f)	d = m – A, A = 25	$d' = \frac{m-A}{C}$, C = 10	fd′
5	10	–20	–2	–20
15	20	–10	–1	–20
25	30	0	0	0
35	20	10	1	20
45	10	20	2	20
	$\Sigma f = 90$			$\Sigma fd' = 0$

Mean $(\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 25 + \frac{0}{90} \times 10 = 25$

CASE 3: Inclusive Class-Intervals *(Classes of type 10-19, 20-29 are given)*: Convert Inclusive Class-Intervals into Exclusive Series.

Marks	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
No. of Students	4	8	4	6	2

Marks (X)	No. of Stud. (f)	Mid-value (m)	d = m – A, A = 34.5	$d' = \frac{m-A}{C}$, C = 10	fd′
9.5 – 19.5	4	14.5	–20	–2	–8
19.5 – 29.5	8	24.5	–10	–1	–8
29.5 – 39.5	4	34.5	0	0	0
39.5 – 49.5	6	44.5	10	1	6
49.5 – 59.5	2	54.5	20	2	4
	$\Sigma f = 24$				$\Sigma fd' = -6$

Mean $(\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 34.5 + \frac{-6}{24} \times 10 = 32$ Marks

CASE 4: Open-End Series *(Lower limit of first class and upper limit of last class not given)*: Missing limits need to be assumed depending on the pattern of class-intervals.

Marks	Less than 40	40 – 50	50 – 60	60 – 70	More than 70
No. of Students	2	8	6	10	4

In the given case, class-intervals are uniform, i.e. 10. So, we can assume that class-intervals of open-end classes are also equal to 10. It means, lower limit of first class is 30 and upper limit of last class will be 80.

Marks (X)	No. of Stud. (f)	Mid value (m)	d = m – A, A = 55	$d' = \frac{m-A}{C}$, C = 10	fd′
30 – 40	2	35	–20	–2	–4
40 – 50	8	45	–10	–1	–8
50 – 60	6	55	0	0	0
60 – 70	10	65	10	1	10
70 – 80	4	75	20	2	8
	$\Sigma f = 30$				$\Sigma fd' = 6$

Mean $(\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 55 + \frac{6}{30} \times 10 = 57$ Marks

CASE 5: Unequal Class-Intervals There is no need to make class-intervals equal, i.e. calculate Mean in usual manner.

X	0 – 5	5 – 10	10 – 20	20 – 30	30 – 50	50 – 60
f	9	3	6	5	5	8

Marks (X)	No. of Stud. (f)	Mid value (m)	d = m – A, A = 25	$d' = \frac{m-A}{C}$, C = 5	fd′
0 – 5	9	2.5	–22.5	–4.5	–40.5
5 – 10	3	7.5	–17.5	–3.5	–10.5
10 – 20	6	15	–10	–2	–12
20 – 30	5	25	0	0	0
30 – 50	5	40	15	3	15
50 – 60	8	55	30	6	48
	$\Sigma f = 36$				$\Sigma fd' = 0$

Mean $(\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 25 + \frac{0}{36} \times 5 = 25$ Marks

Quick Learning – Arithmetic Mean in Special Cases

1. **Cumulative Series ('Less than' or 'More than'):** Convert the cumulative frequency into a simple frequency distribution and then calculate mean in the usual manner.
2. **Mid-Values are given:** Calculate mean in the usual manner without converting the mid-values into class-intervals.
3. **Inclusive Class-Intervals:** Calculate mean in the usual manner without converting the series into an exclusive class-interval series.
4. **Open-end Series:** To calculate mean, missing class limits are assumed, which depends on the pattern of class-intervals of other classes.
5. **Unequal Class-Intervals:** Mean can be determined in the usual manner after calculating the mid-values of each interval.

8.13 PROPERTIES OF ARITHMETIC MEAN

1. ***The sum of deviations of the observations from their arithmetic mean is always zero, i.e., $\Sigma(X-\bar{X})=0$.*** It happens because arithmetic mean is a point of balance, i.e. sum of positive deviations from mean is equal to sum of negative deviations. Due to this property, arithmetic mean is characterised as the centre of gravity.

This can be made clear with the help of an illustration:

X	$(X-\bar{X})$ $(\bar{X}=7)$
3	−4
5	−2
8	+1
12	+5
ΣX = 28	$\Sigma(X-\bar{X})=0$

2. ***The sum of the square of the deviations of the items from their Arithmetic Mean is minimum, i.e., $\Sigma(X-\bar{X})^2$ is minimum.*** The sum is less than the sum of the square of the deviations of the items from any other value.

It is made clear with the following illustration:

X	$(X-\bar{X})$ $\bar{X}=7$	$(X-\bar{X})^2$	X − 8	$(X-8)^2$
3	−4	16	−5	25
5	−2	4	−3	9
8	+1	1	0	0
12	+5	25	+4	16
		$\Sigma(X-\bar{X})^2=46$		$\Sigma(X-8)^2=50$

3. ***Mean of the combined series:*** If the arithmetic mean and number of items of two or more than two related groups are given, then we can compute the combined means of the series as a whole. *{Combined Mean is discussed in detail in 8.14 Section}*

4. ***If each observation of a series is increased or decreased by a constant, say k, then the arithmetic mean of the new series also get increased or decreased by k. i.e., new mean is $\bar{X}-k$.*** *For example,* the arithmetic mean of four items (3, 5, 8, 12) is 7. If 2 is added to each of the four items, then mean of new four items (5, 7, 10, 14) will also increase by 2, i.e. new mean will be 9.

5. ***If all the items in a series are multiplied or divided by a constant, then the mean of these observations also gets multiplied or divided by it.*** *For example,* the arithmetic mean of four items (3, 5, 8, 12) is 7. If each item is multiplied by, say 5, then mean of new four items (15, 25, 40, 60) will also become 5 times of the original mean, i.e. new mean will be 35.

4th and 5th Property

Suppose mean of a series is 30. The resultant mean in the following cases:

Q. 1. When each item of the series is increased by 3.
Ans. New Mean = 30 + 3 = **33**

Q. 2. When each item of the series is decreased by 5.
Ans. New Mean = 30 − 5 = **25**

Q. 3. When each item of the series is multiplied by 2.
Ans. New Mean = 30 × 2 = **60**

Q. 4. When each item of the series is divided by 6.
Ans. New Mean = 30 ÷ 6 = **5**

6. ***If out of arithmetic mean ($\bar{X}$), number of items (N) and total of the values (ΣX), any two values are known, then third value can be easily found out.***

$$\bar{X}=\frac{\Sigma X}{N};\ \text{or}\ \Sigma X=\frac{\bar{X}}{N};\ \text{or}\ N=\frac{\Sigma X}{\bar{X}}$$

On the basis of this property, we can determine the missing items, missing frequency or correct mean, in case of any error.

8.14 COMBINED MEAN

When two or more distributions are given with their number of items and arithmetic means, the combined mean can be calculated by applying the following formula:

$$\bar{X}_{1,2}=\frac{N_1\bar{X}_1+N_2\bar{X}_2}{N_1+N_2}$$

{Where, $\bar{X}_{1,2}$ = Combined Mean; $\bar{X}_1$ = Arithmetic Mean of first distribution; $\bar{X}_2$ = Arithmetic Mean of second distribution; N_1 = Number of items of first distribution; N_2 = Number of items of second distribution}

The aforesaid formula can be extended to more than two distributions in the following form:

$$\bar{X}_{1,2,\ldots\ldots,n}=\frac{N_1\bar{X}_1+N_2\bar{X}_2+\ldots\ldots\ldots\ldots+N_n\bar{X}_n}{N_1+N_2+\ldots\ldots\ldots N_n}$$

The concept of combined mean will be more clear from the following examples.

Example 29. Find out combined mean from the following data:

	Series X_1	Series X_2
Mean	12	20
No. of Items	80	60

Solution:

$$\text{Combined Mean } (\bar{X}_{1,2})=\frac{N_1\bar{X}_1+N_2\bar{X}_2}{N_1+N_2}$$

Given: $\bar{X}_1=12,\ \bar{X}_2=20,\ N_1=80,\ N_2=60$

$$(\overline{X}_{1,2}) = \frac{(80 \times 12) + (60 \times 20)}{80 + 60} = \frac{960 + 1{,}200}{140} = 15.43$$

Ans. Combined Mean = 15.43

Example 30. The average rainfall of a city from Monday to Saturday is 0.3 cms. Due to heavy rainfall on Sunday, the average for the whole week rose to 0.5 cms. How much was the rainfall on Sunday?

Solution:

Consider the rainfall from Monday to Saturday (6 days) as first group and rainfall on Sunday (1 day) as second group.

$$\text{Combined Mean } (\overline{X}_{1,2}) = \frac{N_1\overline{X}_1 + N_2\overline{X}_2}{N_1 + N_2}$$

Given: $N_1 = 6, N_2 = 1, \overline{X}_1 = 0.3, \overline{X}_{1,2} = 0.5$

$$0.5 = \frac{(6 \times 0.3) + (1 \times \overline{X}_2)}{6 + 1}$$

$3.5 = 1.8 + \overline{X}_2$

$\overline{X}_2 = 1.7$

Ans. Rainfall on Sunday = 1.7 cms

Example 31. The average marks of 50 students in class is 5. The pass result of 40 students who took up a class test is given below. Calculate mean marks of 10 students who failed.

Marks	4	5	6	7	8	9
No. of Students	8	10	9	6	4	3

Solution:

$\overline{X}_{1,2} = 5$; Mean of Pass Students $(\overline{X}_1) = ?$, Mean of Fail Students $(\overline{X}_2) = ?$, $N_1 = 40$, $N_2 = 10$

Calculation of Mean Marks of 40 students $(\overline{X}_1)$

Marks (X)	No. of Students (f)	fX
4	8	32
5	10	50
6	9	54
7	6	42
8	4	32
9	3	27
	$\Sigma f = 40$	$\Sigma fX = 237$

$$\overline{X}_1 = \frac{\Sigma fX}{\Sigma f} = \frac{237}{40} = 5.925 \text{ marks}$$

$$\text{Combined Mean } (\overline{X}_{1,2}) = \frac{N_1\overline{X}_1 + N_2\overline{X}_2}{N_1 + N_2}$$

$$5 = \frac{(40 \times 5.925) + (10 \times \overline{X}_2)}{40 + 10}$$

$250 = 237 + 10\overline{X}_2$

$\overline{X}_2 = 1.3$

Ans. Average marks of 10 students who failed = 1.3 marks

Example 32. The mean wage of 100 workers is ₹ 284. The mean wage of 70 workers is ₹ 290. Find the mean wage of remaining 30 workers.

Solution:

$$\text{Combined Mean } (\overline{X}_{1,2}) = \frac{N_1\overline{X}_1 + N_2\overline{X}_2}{N_1 + N_2}$$

Given: $\overline{X}_{1,2} = 284, \overline{X}_1 = 290, N_1 = 70, N_2 = 30$

$$284 = \frac{(70 \times 290) + (30 \times \overline{X}_2)}{70 + 30}$$

$28{,}400 = 20{,}300 + 30\overline{X}_2$

$30\overline{X}_2 = 8{,}100$

$$\overline{X}_2 = \frac{8{,}100}{30} = ₹\,270$$

Ans. Mean wage of 30 workers = ₹ 270

Example 33. The mean age of a combined group of men and women is 30 years. If the mean age of the group of men is 32 and that of the group of women is 27, find out the percentage of men and women in the group.

Solution:

Let x be the percentage of men in the combined group. Therefore, percentage of women = 100 – x.

Given: $\overline{X}_1$ (Men) = 32 years; $\overline{X}_2$ (Women) = 27 years; $\overline{X}_{1,2}$ (Combined) = 30 years

$$\text{Combined Mean } (\overline{X}_{1,2}) = \frac{N_1\overline{X}_1 + N_2\overline{X}_2}{N_1 + N_2}$$

$$30 = \frac{32x + 27(100 - x)}{x + (100 - x)}$$

$32x - 27x = 3{,}000 - 2{,}700$

$5x = 300$ or $x = 60$. It means, men are 60% and women = 100 – 60 = 40%

Ans. Men = 60%; Women = 40%

8.15 CORRECTED MEAN

At times, due to mistake or oversight, certain wrong items may be taken while calculating the arithmetic mean. In such a case, we can directly calculate correct arithmetic mean, without calculating the arithmetic mean from the beginning.

$$\text{Correct } \overline{X} = \frac{\Sigma X(\text{Wrong}) + (\text{Correct value}) - (\text{Incorrect value})}{N}$$

New ΣX = old ΣX + Correct value - Incorrect value

old ΣX = X̄·N.

Steps to Calculate Correct Arithmetic Mean

The steps involved in calculating correct arithmetic mean ($\overline{X}$) are:

1. First of all, Incorrect ΣX is calculated. (We know, $\overline{X} = \frac{\Sigma X}{N}$. So, Incorrect $\Sigma X = N\overline{X}$).
2. From this Incorrect ΣX, subtract wrong or incorrect items and add correct items to get Correct ΣX.
3. Divide Correct ΣX by number of items (N) to get Correct $\overline{X}$, i.e.

$$\text{Correct } \overline{X} = \frac{\text{Correct } \Sigma X}{N}$$

This is illustrated in the following examples.

Example 34. The average weight of a group of 25 boys was calculated to be 52 kg. It was later discovered that one weight was misread as 45 kg instead of 54 kg. Calculate the correct average weight.

Solution:

$\overline{X} = \frac{\Sigma X}{N}$

Or, $\Sigma X = \overline{X} \times N$

Given: $\overline{X} = 52$, N = 25

$\Sigma X = 52 \times 25 = 1,300$

But 1,300 is a wrong value as the weight of one boy was misread as 45 kg instead of 54 kg.

Correct ΣX = 1,300 – Incorrect Item + Correct Item = 1,300 – 45 + 54 = 1,309

Correct Average Height ($\overline{X}$) = $\frac{\Sigma X}{N} = \frac{1,309}{25} = 52.36$ kg

Ans. Correct Average Weight = 52.36 kg

Example 35. The mean salary paid to 1,000 employees of a factory was found to be ₹ 180.4. Later on it was discovered that the wages of two employees were wrongly taken as 297 and 165 instead of 197 and 185. Find the correct mean salary.

Solution:

$\overline{X} = \frac{\Sigma X}{N}$

Or, $\Sigma X = \overline{X} \times N$

Given: $\overline{X} = 180.4$, N = 1,000

$\Sigma X = 180.4 \times 1,000 = 1,80,400$

But 1,80,400 is a wrong value as the wages of two employees were wrongly taken as 297 and 165 instead of 197 and 185.

Corrected ΣX = 1,80,400 – Incorrect Item + Correct Item

Corrected ΣX = 1,80,400 – 297 – 165 + 197 + 185 = 1,80,320

Correct Mean Salary ($\overline{X}$) = $\frac{\Sigma X}{N} = \frac{1,80,320}{1,000}$ = ₹ 180.32

Ans. ₹ 180.32

Example 36. The average marks in statistics of 10 students of a class were 68. A new student took admission with 72 marks whereas two existing students left the college. If the marks of these students were 40 and 39, find the correct average marks.

Solution:

$\overline{X} = \frac{\Sigma X}{N}$

Or, $\Sigma X = \overline{X} \times N$

Given: $\overline{X} = 68$, N = 10

$\Sigma X = 68 \times 10 = 680$

Corrected ΣX = 680 – 40 – 39 + 72 = 673

Correct Average Marks ($\overline{X}$) = $\frac{\Sigma X}{N} = \frac{673}{9}$ = 74.78 marks

Ans. Correct Average Marks = 74.78 marks

Example 37. The average age of a class having 35 students is 14 years. When the age of the class teacher is added to the sum of the ages of the students, the average rises by 0.5 year. What must be the age of the teacher?

Solution:

$\overline{X} = \frac{\Sigma X}{N}$

Or, $\Sigma X = \overline{X} \times N$

Total age of 35 students = 35 × 14 = 490

Total age of students and the teacher together = 36 × 14.5 = 522

Age of teacher = 522 – 490 = 32 years

Ans. Teacher's age = 32 years

Example 38. What will be the new mean, if it is known that for a group of 10 students, scoring an average of 60 marks, the best paper was wrongly marked 80 instead of 75?

Solution:

$\overline{X} = \frac{\Sigma X}{N}$

$\Sigma X = \overline{X}N$

$\overline{X} = 60$, N = 10

$\Sigma X = 60 \times 10 = 600$

Corrected ΣX = 600 – 80 + 75 = 595

Correct Mean ($\overline{X}$) = $\frac{\Sigma X}{N} = \frac{595}{10}$ = 59.5 marks

Ans. New Mean = 59.5 marks

8.16 MERITS AND DEMERITS OF ARITHMETIC MEAN

Merits of Arithmetic Mean

The arithmetic mean is the most widely used measure of central tendency in practice because of the following merits:

1. Simple to Understand and Easy to Compute: The calculation of arithmetic mean requires simple knowledge of addition, multiplication and division of numbers.
   - So, even a layman with elementary knowledge can calculate arithmetic mean.
   - It is also simple to understand the meaning of arithmetic mean, e.g., the value per item or cost per unit, etc.
2. Certainty: Arithmetic mean is rigidly defined by an algebraic formula. Therefore, everyone who computes the average, get the same answer. Arithmetic mean leaves no scope for deliberate prejudice or personal bias.
3. Based on all items: Arithmetic mean takes into account all values into consideration. So, it is considered to be more representative of the distribution.
4. Least affected by fluctuations in sample: Of all the averages, arithmetic mean is least affected by fluctuations of sampling.
   - If the number of items in a series is large, the arithmetic mean provides a good basis of comparison since abnormalities (errors) in one direction are set off against the abnormalities in another direction.
   - Due to this reason, arithmetic average is believed to be a stable measure.
5. Convenient Method of Comparison: Arithmetic Average forms a convenient method of comparison of two or more distributions.
6. Algebraic treatment: Arithmetic mean is capable of further algebraic treatment. It is capable of being treated mathematically and hence, it is widely used in the computation of various other statistical measures such as mean deviation, standard deviation, etc.
7. No arrangement required: The computation of arithmetic mean does not involve the arrangement or grouping of items.

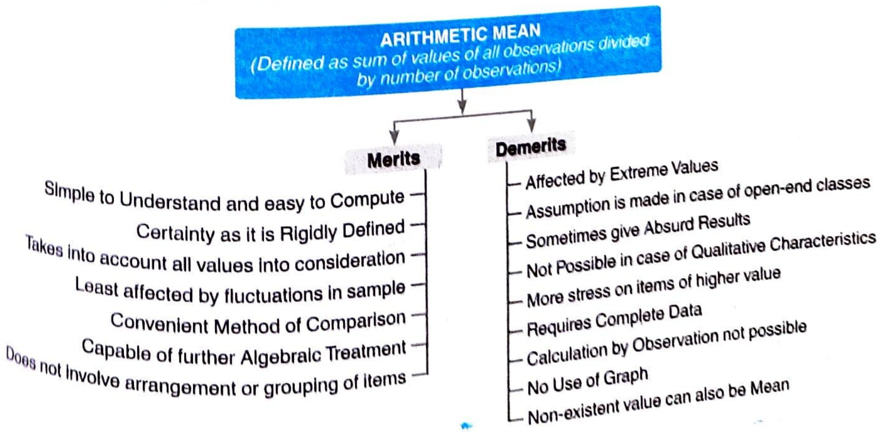
Demerits of Arithmetic Mean

Although, arithmetic mean satisfies most of the properties of an ideal average, it has certain drawbacks and should be used with care. Some demerits of arithmetic mean are:

1. Affected by extreme values: Since arithmetic average is calculated from all the items of a series, it is unduly affected by extreme values (i.e. very small or very large items).

   For example, if monthly income of four persons is 5,000; 7,000; 8,000; and 1,00,000, then their arithmetic mean will be 30,000, which does not represent the data.

2. Assumption in Case of Open-end Classes: In case of open-end classes, the arithmetic mean cannot be calculated unless assumptions are made regarding the magnitude of class-intervals of the open-end classes.
3. Absurd results: Arithmetic mean sometimes gives such results which appear almost absurd. If we have an average of 3.2 children per family for a particular community, obviously the result (average) is absurd as a child cannot be divided into fractions.
4. Not possible in case of qualitative characteristics: Arithmetic mean cannot be computed for a qualitative data; like data on intelligence, honesty, smoking habit, etc. In such cases, median (discussed later) is the only average to be used.
5. More stress on items of higher value: The arithmetic mean gives more importance to higher items of a series as compared to smaller items, i.e. it has an upward bias.
   - If out of five items, four are small, and one item is quite big, then big item will push up the average considerably.
   - But, the reverse is not true. If in series of five items, four have big values and one has small value, the arithmetic average will not be pulled down very much.
6. Complete data required: The arithmetic mean cannot be calculated without all the items of a series. *For example,* if out of 1,000 items, the values of 999 items are known, then arithmetic average cannot be calculated. Other averages like median and mode do not need complete data.
7. Calculation by observation not possible: Arithmetic mean cannot be computed by simply observing the series like median or mode.
8. No Use of Graph: Arithmetic mean cannot be calculated by using graph.
9. Non-existent value as mean: Sometimes, arithmetic average can be a fictitious figure which does not exist in the series. The arithmetic average of 8, 14, 17, and 25 is 16. No items of the series have value of 16.



8.17 WEIGHTED MEAN

Meaning

Weighted Mean refers to the average when different items of a series are given different weights according to their relative importance.

- In the computation of simple arithmetic mean, it is assumed that all the items in the series are of equal importance. However, there are situations, in which values of observations in the series are not of equal importance.
- If all the items are not of equal importance, then simple arithmetic mean will not be a good representative of the given data. Hence, weighting of different items becomes necessary.
- The weights are assigned to different items depending upon their importance, i.e., more important items are assigned more weight.

Computation of Weighted Mean

In calculating the weighted mean, each item of the series is multiplied by its weights and the product so obtained is totalled. This total is divided by the total of weights and the resulting figure is weighted mean.

Let $W_1, W_2......W_n$ be the weights attached to variable values $X_1, X_2.......X_n$ respectively. Then the weighted arithmetic mean, usually denoted by $\overline{X}_W$ is given by:

$$\overline{X}_W = \frac{W_1X_1 + W_2X_2 + ... + W_nX_n}{W_1 + W_2 + ... + W_n}$$

The above formula can be written in short as:

$$\overline{X}_W = \frac{\Sigma WX}{\Sigma W}$$

{Where, $\overline{X}_W$ = Weighted Mean; ΣWX = Sum of the products of the items and their respective weights; ΣW = Sum of the weights}

Steps for Calculating Weighted Mean

1. Denote the variables as X and weights as W.
2. Multiply variables (X) with weights (W) and obtain the total to get ΣWX.
3. Apply the following formula: $\overline{X}_W = \frac{\Sigma WX}{\Sigma W}$

The concept of weighted mean will be clear from the following examples.

Example 39. Calculate the weighted mean of the following data:

Items	10	15	20	25	30	35
Weight	6	9	4	10	5	2

Solution:

Items (X)	Weight (W)	WX
10	6	60
15	9	135
20	4	80
25	10	250
30	5	150
35	2	70
	$\Sigma W = 36$	$\Sigma WX = 745$

$$\overline{X}_W = \frac{\Sigma WX}{\Sigma W} = \frac{745}{36} = 20.69$$

Ans. Weighted Mean = 20.69

Example 40. Calculate weighted mean by weighting each price by the quantity consumed:

Food items	Quantity consumed (in kg)	Price in Rupees (per kg)
Wheat	300	10
Rice	400	20
Sugar	200	15
Potato	500	7

Solution:

Food items	Quantity consumed (in kg) (W)	Price in Rupees (per kg) (X)	WX
Wheat	300	10	3,000
Rice	400	20	8,000
Sugar	200	15	3,000
Potato	500	7	3,500
	$\Sigma W = 1,400$		$\Sigma WX = 17,500$

$$\overline{X}_W = \frac{\Sigma WX}{\Sigma W} = \frac{17,500}{1,400} = 12.5$$

Ans. Weighted mean = 12.5

Example 41. A candidate obtained the following percentage of marks in different subjects in an examination:

Subject	Marks
English	70
Maths	85
Economics	90
Business Studies	80
Accounts	95

Find the weighted Mean if weights are 2, 1, 2, 3, 4 respectively.

Solution:

Subject	Marks X	Weights W	WX
English	70	2	140
Maths	85	1	85
Economics	90	2	180
Business Studies	80	3	240
Accounts	95	4	380
		$\Sigma W = 12$	$\Sigma WX = 1{,}025$

$$\bar{X}_W = \frac{\Sigma WX}{\Sigma W} = \frac{1{,}025}{12} = 85.42$$

Ans. Weighted mean = 85.42 marks

Example 42. Calculate the value of weighted mean from the given details of a college:

Course	Students Appeared	Students Passed
B. Com (H)	200	180
B. Com (P)	400	320
B.A.	700	490
M. Com	300	150

Solution:

Course	Students Appeared (W)	Students Passed	Percentage Pass (X) $\frac{\text{Passed}}{\text{Appeared}} \times 100$	WX
B. Com (H)	200	180	90	18,000
B. Com (P)	400	320	80	32,000
B.A.	700	490	70	49,000
M. Com	300	150	50	15,000
	$\Sigma W = 1{,}600$			$\Sigma WX = 1{,}14{,}000$

$$\bar{X}_W = \frac{\Sigma WX}{\Sigma W} = \frac{1{,}14{,}000}{1{,}600} = 71.25\%$$

Ans. Weighted mean = 71.25 %

Example 43. An examination was held to decide the award of a scholarship. The weights of various subjects were different. The marks obtained by 3 candidates (out of 100 in each subject) are given below:

Subject	Weights	Marks Student A	Student B	Student C
Mathematics	4	60	57	62
Business Studies	3	62	61	67
Economics	2	55	53	60
English	1	67	77	49

Calculate the weighted Arithmetic Mean to award the scholarship.

Solution:

Subject	Weights (W)	Student A Marks (X_A)	WX_A	Student B Marks (X_B)	WX_B	Student C Marks (X_C)	WX_C
Mathematics	4	60	240	57	228	62	248
Business Studies	3	62	186	61	183	67	201
Economics	2	55	110	53	106	60	120
English	1	67	67	77	77	49	49
	10	244	603	248	594	238	618

	Simple Arithmetic Mean	Weighted Mean
Student A	$\bar{X}_A = \frac{\Sigma X_A}{N} = \frac{244}{4} = 61$	$\bar{X}_{WA} = \frac{\Sigma WX_A}{\Sigma W} = \frac{603}{10} = 60.3$
Student B	$\bar{X}_B = \frac{\Sigma X_B}{N} = \frac{248}{4} = 62$	$\bar{X}_{WB} = \frac{\Sigma WX_B}{\Sigma W} = \frac{594}{10} = 59.4$
Student C	$\bar{X}_C = \frac{\Sigma X_C}{N} = \frac{238}{4} = 59.5$	$\bar{X}_{WC} = \frac{\Sigma WX_C}{\Sigma W} = \frac{618}{10} = 61.8$

Ans. From the above calculations, C should get the scholarship as his weighted mean is the highest.

Note: *According to simple arithmetic mean, B should get the scholarship. But all the subjects of examination are not of equal importance. Therefore, weighted mean is to be considered for award of scholarship.*

Example 44. Under what conditions, weighted mean is:

1. Equal to simple arithmetic mean;
2. Greater than simple arithmetic mean;
3. Less than simple arithmetic mean.

Illustrate the answer with the help of an example.

Solution:

1. **Weighted mean is equal to simple arithmetic mean when equal weights are used for all the items in the series or distribution.** This will be clear from the following example:

Values (X)	W	WX
80	2	160
20	2	40
30	2	60
25	2	50
45	2	90
ΣX = 200	ΣW = 10	ΣWX = 400

Simple Arithmetic Mean $(\overline{X}) = \frac{\Sigma X}{N} = \frac{200}{5} = 40$

Weighted Mean $(\overline{X}_W) = \frac{\Sigma WX}{\Sigma W} = \frac{400}{10} = 40$

Simple Arithmetic Mean = Weighted Mean = 40

2. **Weighted mean is greater than simple arithmetic mean when larger weights are assigned to the larger items and smaller weights to the smaller items.** The following example will prove this point:

Values (X)	W	WX
80	5	400
20	1	20
30	3	90
25	2	50
45	4	180
ΣX = 200	ΣW = 15	ΣWX = 740

Simple Arithmetic Mean $(\overline{X}) = \frac{\Sigma X}{N} = \frac{200}{5} = 40$

Weighted Mean $(\overline{X}_W) = \frac{\Sigma WX}{\Sigma W} = \frac{740}{15} = 49.33$

Weighted mean (49.33) is greater than simple arithmetic mean (40).

3. **Weighted mean is less than simple arithmetic mean when smaller weights are assigned to larger figures and larger weights to the smaller items.**

Values (X)	W	WX
80	1	80
20	5	100
30	3	90
25	4	100
45	2	90
ΣX = 200	ΣW = 15	ΣWX = 460

Simple Arithmetic Mean $(\overline{X}) = \frac{\Sigma X}{N} = \frac{200}{5} = 40$

Weighted Mean $(\overline{X}_W) = \frac{\Sigma WX}{\Sigma W} = \frac{460}{15} = 30.67$

Weighted mean (30.67) is less than simple arithmetic mean (40).

FORMULAE AT A GLANCE

1. SIMPLE MEAN **Individual Series** *Direct Method*	$\overline{X} = \frac{\Sigma X}{N}$	$\overline{X}$ = Arithmetic Mean ΣX = Summation of values of Variable X N = Number of observations
Short-cut Method	$\overline{X} = A + \frac{\Sigma d}{N}$	A = Assumed Mean Σd = Sum of deviations of variables from assumed mean
Step Deviation Method	$\overline{X} = A + \frac{\Sigma d'}{N} \times C$	$\Sigma d'$ = Sum of step deviations C = Common Factor
Discrete Series *Direct Method*	$\overline{X} = \frac{\Sigma fX}{\Sigma f}$	ΣfX = Sum of product of Variable (X) and frequencies (f) Σf = Total of frequencies
Shot-cut Method	$\overline{X} = A + \frac{\Sigma fd}{\Sigma f}$	Σfd = Sum of product of deviations (d) and respective frequencies (f)
Step Deviation Method	$\overline{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C$	$\Sigma fd'$ = Sum of product of step deviations (d') and respective frequencies (f)
Continuous Series *Direct Method*	$\overline{X} = \frac{\Sigma fm}{\Sigma f}$	m = Mid-Points Σfm = Sum of product of mid-points (m) and frequencies (f)
Shot-cut Method	$\overline{X} = A + \frac{\Sigma fd}{\Sigma f}$	Σfd = Sum of product of deviations (d) from mid-points with the respective frequencies (f)
Step Deviation Method	$\overline{X} = A + \frac{\Sigma fd'}{\Sigma f} \times C$	$\Sigma fd'$ = Sum of product of step deviations (d') and frequencies (f)
2. Combined Mean	$(\overline{X}_{1,2}) = \frac{N_1\overline{X}_1 + N_2\overline{X}_2}{N_1 + N_2}$	$\overline{X}_{1,2}$ = Combined Mean $\overline{X}_1$ = Arithmetic Mean of first distribution $\overline{X}_2$ = Arithmetic Mean of second distribution N_1 = Number of items of first distribution N_2 = Number of items of second distribution
3. Weighted Mean	$\overline{X}_W = \frac{\Sigma WX}{\Sigma W}$	$\overline{X}_W$ = Weighted Mean ΣWX = Sum of product of items and respective weights ΣW = Sum of the weights

9

MEASURES OF CENTRAL TENDENCY — MEDIAN AND MODE

LEARNING OBJECTIVES

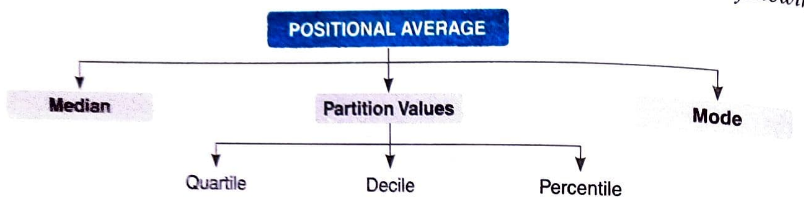
9.1 INTRODUCTION

In the previous chapter, we discussed the concept of simple arithmetic mean and weighted mean, which are mathematical in nature.

- Such mathematical averages deal with those characteristics of a data set which can be directly measured quantitatively.
- However, at times, we need to measure qualitative characteristics of a distribution.
- In such cases, the other measures of the central tendency are *"Positional Averages"*.

Meaning of Positional Average

Positional average determines the position of variables in the series. The positional averages have nothing to do with the sum of the values of the variable. As a result, they are least affected by the extreme items of the series. *The various positional averages are shown in the following chart:*



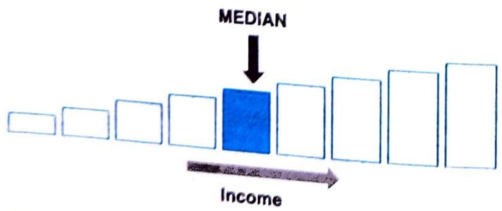
However, the present chapter focuses only on median and mode, in accordance with the CBSE syllabus. *A brief reference of Quartiles is also given as its knowledge is important to understand the concept of 'Quartile Deviation', discussed in the next chapter.*

9.2 MEDIAN

Median may be defined as the middle value in the data set when its elements are arranged in a sequential order, that is, in either ascending or descending order of magnitude. Its value is so located in a distribution that it divides it in half, with 50% items below it and 50% above it.

- It concentrates on the middle or centre of a distribution.
- Median is that positional value of the variable which divides the distribution into two equal parts:
  - One part comprises all values greater than or equal to the median value; and
  - The other part comprises all values less than or equal to it.

In the words of Yule and Kendall, *'The median may be defined as the middle most value of the variable when items are arranged in order of magnitude or as the value such that greater and smaller values occur with equal frequency.'*



Example for Better Understanding: Suppose weight of 5 persons is 55, 62, 60, 59, 70 kg. Now, to calculate the value of median, the first step is to arrange the data in the ascending (or descending) order. Arranging the weights in ascending order, we get: 55, 59, ***60****, 62, 70. The median value is 60 kg as it occupies the middle position.*

Mean Vs Median

- Unlike arithmetic average, median does not take into account the values of all items in a series.
- For example, if the marks of five students are 40, 42, 50, 55 and 60, the median value would be 50. If however the marks of these students were 38, 45, 50, 60 and 70, median would still be 50, though the two series are different in their composition. Due to this reason, median is called a 'positional average'.
- The value of median is the value of the middle item irrespective of all other values. On the other hand, in case of arithmetic average values, of all items are taken into account and that is why, it is a 'mathematical average'.

9.3 COMPUTATION OF MEDIAN

The median can be calculated in the following types of distributions:

1. Individual Series
2. Discrete Series
3. Continuous Series

Individual Series

To calculate median in an individual series, the following steps are needed:

Step 1. Arrange the data in ascending order or descending order

Step 2. Apply the following formula: Median (Me) = Size of $\left[\frac{N+1}{2}\right]^{th}$

{Where, N = Number of items}

Odd and Even Number of Items

- *In case of odd number of items:* Median = Middle item of distribution
- *In case of even number of items:* Median = Average of two middle items.

The following table will make the calculation of median more clear:

Odd Number Series	*Even Number Series*
It refers to a series when number of items (N) in a data happens to be an odd number. **Steps** needed to determine Median (Me): 1. Arrange data in ascending or descending order. 2. Apply formula: Me = Size of $\left[\frac{N+1}{2}\right]^{th}$ item.	It refers to a series when number of items (N) in a data happens to be an even number. **Steps** needed to determine Median (Me): 1. Arrange data in ascending or descending order. 2. Apply formula: Me = Size of $\left[\frac{N+1}{2}\right]^{th}$ item. Now, median will be the average of the values of two items

Example: Marks of five students are 42, 40, 60, 55 and 50. Calculate Median.

Step 1: Marks Arranged in Ascending Order

40
42
50
55
60

Step 2. Me = Size of $\left[\frac{N+1}{2}\right]^{th}$ item

Me = Size of $\left[\frac{5+1}{2}\right]^{th}$ item = Size of 3^{rd} item

Median = **50**

Also refer Examples 1 and 2.

Example: Marks of six students are 42, 40, 60, 55, 50 and 65. Calculate Median.

Step 1: Marks Arranged in Ascending Order

40
42
50
55
60
65

Step 2. Me = Size of $\left[\frac{N+1}{2}\right]^{th}$ item

Me = Size of $\left[\frac{6+1}{2}\right]^{th}$ item = Size of 3.5^{th} item

To get median, sum of 3^{rd} item and 4^{th} item is divided by 2.

Median = $\frac{50+55}{2}$ = 52.5

Also refer Examples 3 and 4.

Example 1. Find out the median from the following data:

120	200	170	800	620	350	375	640	750

Solution:

Calculation of Median

Serial No.	Items arranged in ascending order
1	120
2	170
3	200
4	350
5	375
6	620
7	640
8	750
9	800
N = 9	

Me = Size of $\left[\frac{N+1}{2}\right]^{th}$ item = Size of $\left[\frac{9+1}{2}\right]^{th}$ item = Size of 5^{th} item = 375

Ans. Median = 375. This means that 50% of the items are less than or equal to 375 and 50% of the items are more than or equal to 375.

Example 2. From the following data of the weekly wages (in ₹) of 7 employees, compute the median wage.

500	550	490	450	520	400	375

Solution:

Calculation of Median

Serial No.	Wages arranged in ascending order
1	375
2	400
3	450
4	490
5	500
6	520
7	550
N = 7	

Me = Size of $\left[\frac{N+1}{2}\right]^{th}$ item = Size of $\left[\frac{7+1}{2}\right]^{th}$ item = Size of 4^{th} item = 490

Ans. Median = ₹ 490

Example 3. Given below is the age of some students. Find out the median of their age: 20, 16, 19, 14, 10, 22, 11, 9

Solution:

Calculation of Median

Serial No.	Age arranged in ascending order
1	9
2	10
3	11
4	14
5	16
6	19
7	20
8	22
N = 8	

The number of items is even, i.e. 8.

Me = Size of $\left[\frac{N+1}{2}\right]^{th}$ item = Size of $\left[\frac{8+1}{2}\right]^{th}$ item = Size of 4.5^{th} item

To get median, the sum of 4^{th} item and 5^{th} item is divided by 2.

Median = $\frac{\text{Size of } 4^{th} \text{ item} + \text{Size of } 5^{th} \text{ item}}{2} = \frac{14+16}{2} = 15$

Ans. Median = 15 years

Example 4. Calculate median from the following data: 245, 230, 265, 236, 220, 250

Solution:

Arranging these observations in ascending order of magnitude, we get: 220, 230, 236, 245, 250, 265.
The number of items is even, i.e. 6.

$$Me = \text{Size of } \left[\frac{N+1}{2}\right]^{th} \text{ item} = \text{Size of } \left[\frac{6+1}{2}\right]^{th} \text{ item} = \text{Size of } 3.5^{th} \text{ item}$$

To get median, the sum of 3^{rd} item and 4^{th} item is divided by 2.

$$\text{Median} = \frac{\text{Size of } 3^{rd} \text{ item} + \text{Size of } 4^{th} \text{ item}}{2} = \frac{236+245}{2} = 240.5$$

Ans. Median = 240.5

Discrete Series

In a discrete series, the values of the variable are given along with their frequencies.

Steps to Calculate Median

The steps involved are:

Step 1. Arrange the frequency distribution either in ascending or descending order;

Step 2. Denote variables (items) as X and frequency as f;

Step 3. Calculate the cumulative frequencies (c.f.);

Step 4. Find the Median item as: (Me) = Size of $\left[\frac{N+1}{2}\right]^{th}$ items

{Where Me = Median and N = Total of frequency}

Step 5. Find the Value of $\left[\frac{N+1}{2}\right]^{th}$ items. It can be found by first locating the cumulative frequency which is equal to $\left[\frac{N+1}{2}\right]^{th}$ or next higher to it and then determining the value corresponding to it. This will be the value of the median.

This can be made clear with the help of Examples 5, 6 and 7.

Example 5. Calculate the median from the following data:

Size (X)	Frequency (f)
3	2
4	1
5	3
6	7
7	4

Solution:

Size (X)	Frequency (f)	Cumulative frequency
3	2	2 = 2
4	1	2 + 1 = 3
5	3	3 + 3 = 6
6	7	6 + 7 = **13 (Median Group)**
7	4	13 + 4 = 17
	N = Σf = 17	

$$Me = \text{Size of } \left[\frac{N+1}{2}\right]^{th} \text{ item} = \text{Size of } \left[\frac{17+1}{2}\right]^{th} \text{ item} = \text{Size of } 9^{th} \text{ item}$$

Since 9^{th} item falls in the cumulative frequency 13 and the size against this cumulative frequency is 6. Therefore, median is 6.

Ans. Median = 6

Example 6. Locate median of the following frequency distribution:

X	5	10	15	20	25	30	35	40
f	7	14	18	36	51	54	52	20

Solution:

X	f	c.f.
5	7	7
10	14	21
15	18	39
20	36	75
25	51	126
30	54	**180 (Median Group)**
35	52	232
40	20	252
	N = Σf = 252	

$$Me = \text{Size of } \left[\frac{N+1}{2}\right]^{th} \text{ item} = \text{Size of } \left[\frac{252+1}{2}\right]^{th} \text{ item} = \text{Size of } 126.5^{th} \text{ item}$$

Since 126.5^{th} item falls in the cumulative frequency of 180 and the size against this cumulative frequency is 30. Therefore, median is 30.

Ans. Median = 30

Example 7. Calculate median from the following series:

X	14	12	10	13	11
f	3	18	3	12	12

Solution:

We first arrange the data in ascending order and in terms of cumulative frequency distribution as under:

X	f	c.f.
10	3	3
11	12	15
12	18	33 (Median Group)
13	12	45
14	3	48
	N = Σf = 48	

$$Me = \text{Size of} \left[\frac{N+1}{2}\right]^{th} \text{item} = \text{Size of} \left[\frac{48+1}{2}\right]^{th} \text{item} = \text{Size of } 24.5^{th} \text{ item}$$

Since 24.5^{th} item falls in the cumulative frequency of 33 and the size against this cumulative frequency is 12. Therefore, median is 12.

Ans. Median = 12

Continuous Series

In case of continuous series, median cannot be located straight-forward. In this case, median lies in between lower and upper limit of a class-interval. *To get the exact value of the median, we have to interpolate (estimate) median with the help of a formula.*

Steps to Calculate Median

The steps involved are:

Step 1. Arrange the data in ascending or descending order.

Step 2. Calculate the cumulative frequencies (c.f.).

Step 3. Find the Median item as: (Me) = Size of $\left[\frac{N}{2}\right]^{th}$ item.

{Where Me = Median and N = Total of frequency}

Step 4. By inspecting cumulative frequencies, find out c.f. which is either equal to or just greater than this.

Step 5. Find the class corresponding to cumulative frequency equal to $\frac{N}{2}$ or just greater than this. This class is called median class.

Step 6. Apply the following formula: $Me = l_1 + \dfrac{\frac{N}{2} - c.f.}{f} \times i$

*{Where, **Me** = Median; l_1 = Lower limit of the median class; **c.f.** = Cumulative frequency of the class preceding the median class; **f** = Simple frequency of the median class; **i** = Class-interval of the median group or class}*

Like mean, in median also, we have to assume that value in each class is uniformly distributed in the class-interval.

Be Attentive in Continuous Series

- In continuous series, median lies in a class-interval, i.e. between lower and upper limit of a class-interval.
- For calculating the exact value of median, it is assumed that the variable is continuous and there is orderly and evenly distribution of items within each class.
- When first class becomes Median Class, then c.f. will be zero and other process for calculation of median will remain the same.
- While computing the value of median, the middle item is $\left[\frac{N}{2}\right]^{th}$ item and not $\left[\frac{N+1}{2}\right]^{th}$ item.
- Median will have 50 percent of the frequencies on one side and the other 50 per cent on the other side.

Example 8. Find the median for the following data:

X	0–10	10–20	20–30	30–40	40–50
f	3	4	2	7	9

Solution:

X	f	c.f.
0–10	3	3
10–20	4	7
20–30	2	9 (c.f.)
(l_1) 30–40	7 (f)	16 Median Class
40–50	9	25
	N = Σf = 25	

$$\text{Median} = \frac{N}{2} = \frac{25}{2} = 12.5^{th} \text{ item}$$

12.5^{th} item lies in the group 30–40

$l_1 = 30$, c.f. = 9, f = 7, i = 10

By applying formula:

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 30 + \frac{12.5 - 9}{7} \times 10 = 35$$

Ans. Median = 35

Example 9. From the following figures, find out median:

Marks	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90
No. of Students	15	21	35	52	49	17	3	1

Solution:

Marks (X)	No. of Students (f)	c.f.
10–20	15	15
20–30	21	36
30–40	35	71 (c.f.)
(l_1) 40–50	52 (f)	123 Median Class
50–60	49	172
60–70	17	189
70–80	3	192
80–90	1	193
	N = Σf = 193	

$Me = \frac{N}{2} = \frac{193}{2} = 96.5^{th}$ item

96.5^{th} item lies in the group 40–50

$l_1 = 40$, c.f. = 71, f = 52, i = 10

By applying formula:

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 40 + \frac{96.5 - 71}{52} \times 10 = 44.90$$

Ans. Median = 44.90 marks. This means that 50% of the students are getting less than or equal to 44.90 marks and 50% of the students are getting more than or equal to this marks.

Example 10. Calculate the median of the following distribution which gives the marks obtained by students in a certain examination.

Marks	40–50	30–40	20–30	10–20	0–10
No. of Students	2	7	12	9	1

Solution:

After arranging the data in ascending order and calculating cumulative frequency, we get the following table:

Marks (X)	No. of Students (f)	c.f.
0–10	1	1
10–20	9	10 (c.f.)
(l_1) 20–30	12 (f)	22 Median Class
30–40	7	29
40–50	2	31
	N = Σf = 31	

$Me = \frac{N}{2} = \frac{31}{2} = 15.5^{th}$ item

15.5^{th} item lies in the group 20–30

$l_1 = 20$, c.f. = 10, f = 12, i = 10

By applying formula:

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 20 + \frac{15.5 - 10}{12} \times 10 = 24.58 \text{ Marks}$$

Ans. Median = 24.58 marks

Example 11. Calculate the value of median from the following frequency distribution.

Marks (X)	0–10	10–20	20–30	30–40	40–50
No. of Students (f)	8	30	40	12	10

Solution:

Marks (X)	No. of Students (f)	c.f.
0–10	8	8
10–20	30	38 (c.f.)
(l_1) 20–30	40 (f)	78 Median Class
30–40	12	90
40–50	10	100
	N = Σf = 100	

$Me = \frac{N}{2} = \frac{100}{2} = 50^{th}$ item

50^{th} item lies in the group 20–30

$l_1 = 20$, c.f. = 38, f = 40, i = 10

By applying formula:

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 20 + \frac{50 - 38}{40} \times 10 = 23$$

Ans. Median = 23 Marks

9.4 MEDIAN IN SPECIAL CASES

The calculation process of Median is different under some special circumstances. Let us discuss these special cases: