Cumulative Series ('Less than' or 'More than')

When the data is given in the form of *"Less than"* or *"More than"* for all items in the series, then **such data has to be converted into a simple frequency distribution,** in order to find out the frequency of the median class. Once it is done, the rest of the procedure is the same as in any other continuous series.

Examples 12, 13 and 14 would illustrate the calculation of median in 'less than' and 'more than' series.

Example 12. Calculate the median from the following data:

Marks	No. of Students
Less than 5	4
Less than 10	10
Less than 15	20
Less than 20	30
Less than 25	55
Less than 30	77
Less than 35	95
Less than 40	100

Solution:

Since we are given the cumulative frequencies, we first find the simple frequency.

Marks (X)	No. of Students (f)	c.f.
0–5	4	4
5–10	6	10
10–15	10	20
15–20	10	**30 (c.f.)**
(l_1) 20–25	**25 (f)**	55 **Median Class**
25–30	22	77
30–35	18	95
35–40	5	100
	N = Σf = 100	

$$Me = \frac{N}{2} = \frac{100}{2} = 50^{th} \text{ item}$$

50^{th} item lies in the group 20–25

$l_1 = 20$, c.f. = 30, f = 25, i = 5

By applying formula:

$$Me = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 20 + \frac{50 - 30}{25} \times 5 = 24$$

Ans. Median = 24

Example 13. Find out the median for the following data:

Age (in years)	No. of Persons
10–20	8
10–30	32
10–40	54
10–50	58
10–60	66
10–70	80

Solution:

In the given example, the data is given in the form of cumulative series. So, it will be first converted into simple series to find the frequency of the median class.

Age in years (X)	No. of Persons (f)	c.f.
10–20	8	8
20–30	24	**32 (c.f.)**
(l_1) 30–40	**22 (f)**	54 **Median Class**
40–50	4	58
50–60	8	66
60–70	14	80
	N = Σf = 80	

$$Me = \frac{N}{2} = \frac{80}{2} = 40^{th} \text{ item}$$

40^{th} item lies in the group 30–40

$l_1 = 30$, c.f. = 32, f = 22, i = 10

By applying formula:

$$Me = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 30 + \frac{40 - 32}{22} \times 10 = 33.63 \text{ years}$$

Ans. Median = 33.63 years

Example 14. Find the median of the following data:

Age in years (Greater than)	0	10	20	30	40	50	60	70
No. of Persons	230	218	200	165	123	73	28	8

Solution:

Note that it is 'more than' type frequency distribution. *We will first convert the cumulative frequencies into* simple frequencies.

Age (in yrs)	No. of Persons (f)	c.f.
0–10	12	12
10–20	18	30
20–30	35	65
30–40	42	**107 (c.f.)**

(l_1) 40–50	50 (f)	157 **Median Class**
50–60	45	202
60–70	20	222
70–80	8	230
	N = Σf = 230	

$$Me = \frac{N}{2} = \frac{230}{2} = 115^{th} \text{ item}$$

115th item lies in the group 40–50

$l_1 = 40$, c.f. = 107, f = 50, i = 10

By applying formula:

$$Me = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 40 + \frac{115 - 107}{50} \times 10 = 41.6 \text{ year}$$

Ans. Median = 41.6 years

Mid-Values are given

When the mid-values of class-intervals are given, then the class-intervals are found, i.e. *to calculate median, we need to first convert it into continuous series.*

Steps to convert Mid-value Series to Continuous Series

Step 1: First of all, calculate the difference between the two mid-values.

Step 2: Then, half of the difference is subtracted and added to each mid-value to find the lower and upper limits respectively of the class-intervals.

Refer Example 15 for better understanding.

Example 15. Compute median from the following data:

Mid-Points	115	125	135	145	155	165	175	185	195
Frequency	6	25	48	72	116	60	38	22	3

Solution:

In the given example, we are given the mid-values. We need to first convert it into continuous series.

Step 1: The difference between the two mid-values is 10.

Step 2: Half of the difference is: $\frac{10}{2} = 5$. Now, 5 is reduced and added to each mid-value to determine the lower limit and upper limit.

It is shown in the following table:

Computation of Median

Marks	f	c.f.
110–120	6	6
120–130	25	31

130–140	48	79
140–150	72	151 (c.f.)
(l_1) 150–160	116 (f)	267 **Median Class**
160–170	60	327
170–180	38	365
180–190	22	387
190–200	3	390
	N = Σf = 390	

$$Me = \frac{N}{2} = \frac{390}{2} = 195^{th} \text{ item}$$

195th item lies in the group 150–160

$l_1 = 150$, c.f. = 151, f = 116, i = 10

By applying formula:

$$Me = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 150 + \frac{195 - 151}{116} \times 10 = 153.79$$

Ans. Median = 153.79

Inclusive Class-Intervals

While calculating median in a continuous series with inclusive class-intervals, ***it is necessary to convert the series into an exclusive class-interval series.***

Steps to convert Inclusive Series into an Exclusive Series

Step 1. Find the difference between the upper limit of a class-interval and lower limit of the next class-interval.

Step 2. Add half of this difference to the upper limit of each class-interval and subtract remaining half from the lower limit of each class-interval. This procedure fills up the gap between two classes and thereby we get the exclusive classes.

This will be clear from Example 16.

Example 16. Compute median from the following data:

Daily Wages (₹)	Frequency
110–119	15
100–109	40
90–99	45
80–89	60
70–79	50
60–69	40
50–59	15

Solution:

This is a case of inclusive class-intervals. To calculate median, it should be made exclusive and arranged in the ascending order, as follows:

Daily Wages (₹)	Frequency (f)	c.f.
49.5–59.5	15	15
59.5–69.5	40	55
69.5–79.5	50	**105 (c.f.)**
(l_1) 79.5–89.5	**60 (f)**	165 **Median Class**
89.5–99.5	45	210
99.5–109.5	40	250
109.5–119.5	15	265
	N Σf = 265	

$$Me = \frac{N}{2} = \frac{265}{2} = 132.5^{th} \text{ item}$$

132.5th item lies in the group 79.5–89.5

$l_1 = 79.5$, c.f. = 105, f = 60, i = 10

By applying formula:

$$Me = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 79.5 + \frac{132.5 - 105}{60} \times 10 = ₹\,84.08$$

Ans. Median = ₹ 84.08

Open-End Series

In case of open-end classes, the lower limit of the first class and upper limit of the last class is not given. **Median is known to be the best average in open-end class-interval series.** *In this case, there is no need to complete the class-interval and formula also remains the same.*

Example 17 would illustrate the point.

Example 17. Calculate the value of median from the following distribution:

Marks (X)	Below 10	10–20	20–30	30–40	40 and Above
No. of Students (f)	3	13	18	11	5

Solution:

The given data consist of open-end classes. However, to calculate the median, there is no need to complete the class-interval.

Marks (X)	No. of Students (f)	c.f.
Below 10	3	3
10–20	13	**16 (c.f.)**
(l_1) 20–30	**18 (f)**	34 **Median Class**
30–40	11	45
40 and Above	5	50
	N Σf = 50	

$$\text{Median} = \frac{N}{2} = \frac{50}{2} = 25^{th} \text{ item}$$

25th item lies in the group 20–30

$l_1 = 20$, c.f. = 16, f = 18, i = 10

By applying formula:

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 20 + \frac{25 - 16}{18} \times 10 = 25 \text{ Marks}$$

Ans. Median = 25 Marks

Unequal Class-Intervals

When the class-intervals are unequal, there is no need to make the class-intervals equal. The frequencies need not be adjusted and the same formula will be applied as discussed before. *This will be clear from the following example.*

Example 18. Calculate the median of the following distribution of data:

Class-interval	0–10	10–30	30–60	60–80	80–90
Frequency	5	15	30	8	2

Solution:

In this question, the class intervals are unequal. However, to calculate median, there is no need to make class-intervals equal.

Class-interval (X)	Frequency (f)	c.f.
0–10	5	5
10–30	15	**20 (c.f.)**
(l_1) 30–60	**30 (f)**	50 **Median Class**
60–80	8	58
80–90	2	60
	N Σf = 60	

$$\text{Median} = \frac{N}{2} = \frac{60}{2} = 30^{th} \text{ item}$$

30th item lies in the group 30–60

$l_1 = 30$, c.f. = 20, f = 30, i = 30

By applying formula:

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 30 + \frac{30 - 20}{30} \times 30 = 40$$

Ans. Median = 40

SUMMARY OF MEDIAN IN SPECIAL CASES

CASE 1a: Cumulative Frequency Distribution *(Less Than Series)*: Convert it into Simple Frequency Distribution and then calculate Median in usual manner.

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50
Students	3	7	9	16	25

Marks (X)	Students (f)	c.f.
0 – 10	3	3
10 – 20	4	7
20 – 30	2	9
30 – 40	7	16
40 – 50	9	25
	N = Σf = 25	

$Me = \frac{N}{2} = \frac{25}{2} = 12.5^{th}$ item; 12.5^{th} item lies in group 30 – 40

$l_1 = 30$ c.f. = 9 f = 7 i = 10

$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 30 + \frac{12.5 - 9}{7} \times 10 =$ **35 Marks**

CASE 1b: Cumulative Frequency Distribution *(More Than Series)*: Convert it into Simple Frequency Distribution and then calculate Median in usual manner.

Marks	More than 10	More than 20	More than 30	More than 40	More than 50
Students	30	24	16	11	4

Marks (X)	Students (f)	c.f.
10 – 20	6	6
20 – 30	8	14
30 – 40	5	19
40 – 50	7	26
50 – 60	4	30
	N = Σf = 30	

$Me = \frac{N}{2} = \frac{30}{2} = 15^{th}$ item; 15^{th} item lies in group 30 – 40

$l_1 = 30$ c.f. = 14 f = 5 i = 10

$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 30 + \frac{15 - 14}{5} \times 10 =$ **32 Marks**

CASE 2: Mid-Values are Given: When Mid-points are given, then convert such mid-values into regular Class-Intervals and then calculate Median in usual manner.

Mid-Points	5	15	25	35	45
Frequency	10	20	30	20	10

Class-Intervals (X)	Frequency (f)	c.f.
0 – 10	10	10
10 – 20	20	30
20 – 30	30	60
30 – 40	20	80
40 – 50	10	90
	N = Σf = 90	

$Me = \frac{N}{2} = \frac{90}{2} = 45^{th}$ item; 45^{th} item lies in group 20 – 30

$l_1 = 20$ c.f. = 30 f = 30 i = 10

$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 20 + \frac{45 - 30}{30} \times 10 =$ **25**

CASE 3: Inclusive Class-Intervals *(Classes of type 10-19, 20-29 are given)*: Convert Inclusive Class-Intervals into Exclusive Series.

Class-Intervals	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
Frequency	3	9	8	7	13

Class-Intervals (X)	Frequency (f)	c.f.
9.5 – 19.5	3	3
19.5 – 29.5	9	12
29.5 – 39.5	8	20
39.5 – 49.5	7	27
49.5 – 59.5	13	40
	N = Σf = 40	

$Me = \frac{N}{2} = \frac{40}{2} = 20^{th}$ item; 20^{th} item lies in group 29.5 – 39.5

$l_1 = 29.5$ c.f. = 12 f = 8 i = 10

$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 29.5 + \frac{20 - 12}{8} \times 10 = 39.5$

CASE 4: Open-End Series *(Lower limit of first class and upper limit of last class not given)*: There is no need to find missing limits, i.e. calculate Median in usual manner.

Class-Intervals	Less than 40	40 – 50	50 – 60	60 – 70	More than 70
Frequency	4	7	6	5	6

Class-Intervals (X)	Frequency (f)	c.f.
Less than 40	4	4
40 – 50	7	11
50 – 60	6	17
60 – 70	5	22
More than 70	6	28
	N = Σf = 28	

$Me = \frac{N}{2} = \frac{28}{2} = 14^{th}$ item; 14^{th} item lies in group 50 – 60

$l_1 = 50$ c.f. = 11 f = 6 i = 10

$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 50 + \frac{14 - 11}{6} \times 10 = 55$

CASE 5: Unequal Class-Intervals There is no need to make class-intervals equal, i.e. calculate Median in usual manner.

X	0 – 5	5 – 10	10 – 20	20 – 30	30 – 50	50 – 60
f	3	5	10	9	4	3

Class-Intervals (X)	Frequency (f)	c.f.
0 – 5	3	3
5 – 10	5	8
10 – 20	10	18
20 – 30	9	27
30 – 50	4	31
50 – 60	3	34
	N = Σf = 34	

$Me = \frac{N}{2} = \frac{34}{2} = 17^{th}$ item; 17^{th} item lies in group 10 – 20

$l_1 = 10$ c.f. = 8 f = 10 i = 10

$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 10 + \frac{17 - 8}{10} \times 10 = 19$

Calculation of Missing Frequencies

When one or more than one frequency is missing, then it is possible to find out the missing frequency.

Steps to Determine Missing Frequency

Step 1. Represent missing frequencies by f_1 or f_2 as the case may be.

Step 2. Apply the formula for calculation of median. In this process, we get an equation which gives us the missing frequencies.

Examples 19 and 20 would clarify the procedure.

Example 19. The following table gives the distribution of monthly salary of 900 employees However, the frequencies of the classes 40–50 and 60–70 are missing. If the median of the distribution is ₹59.25, find the missing frequencies.

Salaries(₹ in '000)	30–40	40–50	50–60	60–70	70–80
No. of Employees	120	?	200	?	185

Solution:

Let f_1 and f_2 be the frequencies of the classes 40 – 50 and 60 – 70 respectively.

Salaries (₹ in '000) (X)	No. of Employees (f)	c.f.
30–40	120	120
40–50	f_1	$120 + f_1$
50–60	200	$320 + f_1$
60–70	f_2	$320 + f_1 + f_2$
70–80	185	900
	N Σf = 900	

$$\text{Median} = \frac{N}{2} = \frac{900}{2} = 450^{th} \text{ item}$$

450^{th} item lies in the group 50–60 (Given median = 59.25)

$l_1 = 50$, c.f. $= 120 + f_1$, $f = 200$, $i = 10$

$$\text{Median} = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i$$

$$59.25 = 50 + \frac{450 - (120 + f_1)}{200} \times 10$$

$$59.25 = 50 + \frac{450 - (120 + f_1)}{200 \div 10}$$

$9.25 \times 20 = 330 - f_1$

$f_1 = 145$

From summation of frequencies, we have:

$120 + f_1 + 200 + f_2 + 185 = 900$

Putting the value of f_1, we get:

$120 + 145 + 200 + f_2 + 185 = 900$

i.e. $f_2 = 250$

Ans. Frequency of class 40 – 50 (f_1) = 145; Frequency of class 60 – 70 (f_2) = 250

Example 20. An incomplete distribution is given below:

Marks	10–20	20–30	30–40	40–50	50–60	60–70	70–80	Total
No. of Students	24	60	?	130	?	50	36	458

You are given that the median value is 47. Using the median formula, fill up missing frequencies.

Solution:

Let f_1 and f_2 be the frequencies of the classes 30 – 40 and 50 – 60 respectively.

Marks (X)	No. of Students (f)	c.f.
10–20	24	24
20–30	60	84
30–40	f_1	$84 + f_1$
40–50	130	$214 + f_1$
50–60	f_2	$214 + f_1 + f_2$
60–70	50	$264 + f_1 + f_2$
70–80	36	458
	N Σf = 458	

$$\text{Median} = \frac{N}{2} = \frac{458}{2} = 229^{th} \text{ item}$$

229th item lies in the group 40 – 50 (Given median = 47)

$l_1 = 40$, c.f. $= 84 + f_1$, $f = 130$, $i = 10$

$$Me = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i$$

$$47 = 40 + \frac{229 - (84 + f_1)}{130} \times 10$$

$$47 = 40 + \frac{229 - (84 + f_1)}{130 \div 10}$$

$7 \times 13 = 145 - f_1$

$f_1 = 54$

From summation of frequencies, we have:

$24 + 60 + f_1 + 130 + f_2 + 50 + 36 = 458$

Putting the value of f_1, we get:

$24 + 60 + 54 + 130 + f_2 + 50 + 36 = 458$

i.e. $f_2 = 104$

Ans. Frequency of class 30–40 (f_1) = 54; Frequency of class 50–60 (f_2) = 104

9.5 GRAPHIC LOCATION OF MEDIAN

Median can be easily located graphically with help of Ogives (cumulative frequency curve). This can be done with the help of any of the two methods: **(i) 'Less than' and 'More than' Ogive Method; (ii) 'Less than' or 'More than' Ogive Method.**

'Less than' and 'More than' Ogive Method

Step 1. Draw two ogives (one 'less than' and one 'more than') from the given data.

Step 2. From the point of intersection of the two ogives, draw a line parallel to the Y-axis. The point where the line cuts the X-axis, is the Median value.

The following example will make this method more clear.

Example 21. Determine the median graphically from the data given below:

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of Students	3	9	18	30	18	12

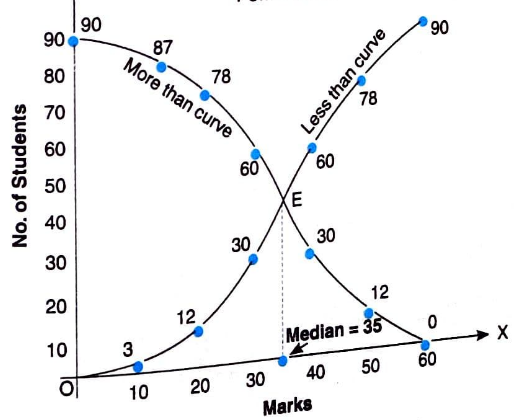
Solution:

In order to calculate median by 'Less than' and 'More than' ogive method, we have to convert the series in cumulative frequency of 'less than' and 'more than' series.

Marks	No. of Students	Marks	No. of Students
Less than 10	3	More than 0	90
Less than 20	12	More than 10	87
Less than 30	30	More than 20	78
Less than 40	60	More than 30	60
Less than 50	78	More than 40	30
Less than 60	90	More than 50	12

On the basis of tables of 'less than' and 'more than', two Ogive curves are drawn:

Graphic Location of Median
'Less than' and 'More than' Ogive Method
Scale: 1 Cm = 10 Marks on X-axis
1 Cm = 10 Students on Y-axis



From the point of intersection (point E), a perpendicular (dotted line in the figure) is drawn on the X-axis. The dotted line cuts the X-axis at 35. Hence the median is 35 marks.

Ans. Median = 35 Marks

'Less than' or 'More than' Ogive Method

In this method, the frequency distribution is converted into either a 'less than' or 'more than' cumulative series, so as to draw the Ogive. The median is determined from the Ogive so drawn.

Step 1. *Draw only one ogive:* Either by 'less than' method or by 'more than' method.

Step 2. Plot the values of the variable on X-axis and the cumulated values (less than) on the Y-axis.

Step 3. Find the Median item as: (Me) = Size of $\left[\frac{N}{2}\right]^{th}$ item

(Where Me = Median and N = Total of frequency)

Step 4. Locate the median item on the Y-axis and from this draw a line parallel to the X-axis to intersect the ogive.

Step 5. Draw a perpendicular line from this point of intersection on the X-axis. The point where the line cuts the X-axis, is the Median value.

Let us understand this with the help of Example 22 ('less than' Ogive) and Example 23 ('more than' Ogive).

Example 22. Determine the value of median graphically by 'less than' ogive with the information given in *Example 21.*

Solution:

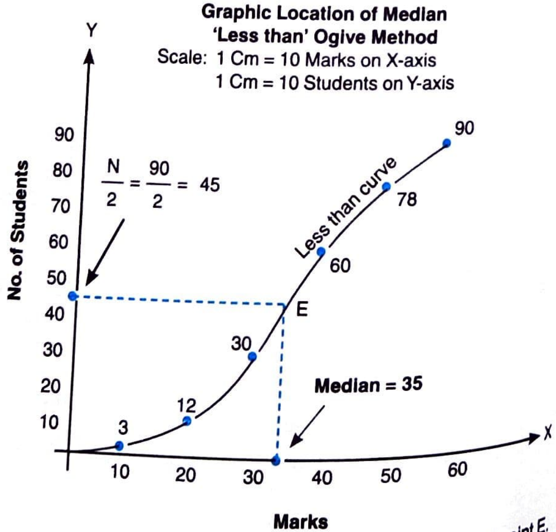
In order to calculate median by 'Less than' ogive method, we have to convert the series in cumulative frequency of 'less than' series.

Marks	No. of Students
Less than 10	3
Less than 20	12
Less than 30	30
Less than 40	60
Less than 50	78
Less than 60	90

On the basis of table of 'less than', one Ogive curve is drawn:

$$Me = \frac{N}{2} = \frac{90}{2} = 45^{th} \text{ item}$$

Graphic Location of Median
'Less than' Ogive Method
Scale: 1 Cm = 10 Marks on X-axis
1 Cm = 10 Students on Y-axis



Locating 45 on the Y-axis and a parallel line from 45 (dotted line in the figure) intersects the ogive at point E. Now, a perpendicular line drawn from point E cuts the X-axis at 35. Hence the median is 35 marks.

Ans. Median = 35 Marks

Example 23. Determine the value of median graphically by 'more than' ogive with the information given in *Example 21.*

Solution:

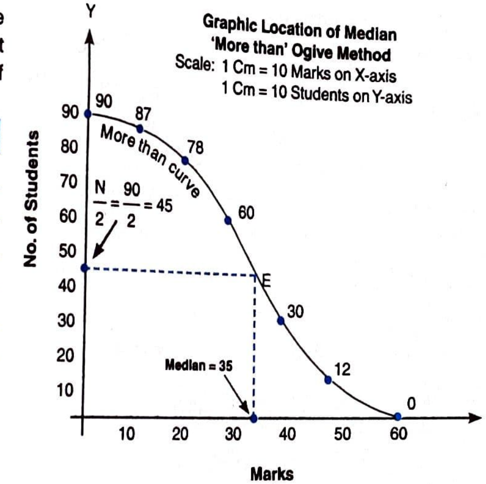
In order to calculate median by 'More than' ogive method, we have to convert the series in cumulative frequency of 'more than' series.

Marks	No. of Students
More than 0	90
More than 10	87
More than 20	78
More than 30	60
More than 40	30
More than 50	12

On the basis of table of 'more than', one Ogive curve is drawn:

$$Me = \frac{N}{2} = \frac{90}{2} = 45^{th} \text{ item}$$

Graphic Location of Median
'More than' Ogive Method
Scale: 1 Cm = 10 Marks on X-axis
1 Cm = 10 Students on Y-axis



Locating 45 on the Y-axis and a parallel line from 45 (dotted line in the figure) intersects the ogive at point E. Now, a perpendicular line drawn from point E cuts the X-axis at 35. Hence the median is 35 marks.

Ans. Median = 35 Marks

9.6 PROPERTIES OF MEDIAN

1. ***The sum of deviations of items from median, ignoring signs, is the minimum.***

   For example, the median of 4, 6, 8, 10, 12 is 8. Now, deviations from 8 (ignoring signs) are 4, 2, 0, 2, 4. The total of these deviations is 12. This total is smaller than the one obtained if deviations are taken from any other value. If deviations are taken from 7, the deviations ignoring signs would be 3, 1, 1, 3, 5 and the total 13. This property implies that median is centrally located.

2. ***Median is a positional average*** and hence it is not influenced by the extreme values.

9.7 MEAN VS MEDIAN

1. Ease in Calculations: Median is easier to calculate as compared to mean.
2. Fluctuations in Sample: The general fluctuations of sampling affect the median to a greater extent than the mean (however, at times mean might be affected to a greater extent by such fluctuations than the median).
3. Algebraic Treatment: Mean is definitely superior to median in terms of further algebraic treatment. It is possible to find out the combined mean, but not the combined median.
4. Open-end classes: Mean cannot be determined in case of open-end distribution, whereas, median can be easily calculated.

5. Effect of Extreme values: Median may be more representative than the arithmetic average due to the fact that it is not affected by the values of extreme items.

6. Graphic presentation: The value of median can be determined graphically, whereas the value of mean cannot be graphically ascertained.

9.8 MERITS AND DEMERITS OF MEDIAN

Merits of Median

1. Simplicity: Median is easy to calculate and simple to understand. In many situations, the median can be located simply by inspection.
2. Ideal average: Median is defined rigidly, i.e. median has definite and certain value.
3. Graphic presentation: The value of median can also be determined graphically with the help of ogive curves.
4. Unaffected by extreme values: The extreme values in the data set do not affect the calculation of the median value.

   For example, median of 10, 20, 30, 40 and 150 would be 30, whereas the mean will be 50. So, median in such cases is a better average.

5. Possible even in case of incomplete data: Median can be calculated even when the data is incomplete. *For example*, in case of irregular class-interval or open-end distribution, median can be easily calculated.
6. Appropriate for qualitative data: Median can be used to deal with qualitative characteristics which cannot be measured quantitatively.

   For example, it is not possible to measure intelligence quantitatively. However, it is possible to locate an individual having average intelligence by arraying a group of persons in ascending or descending order of intelligence.

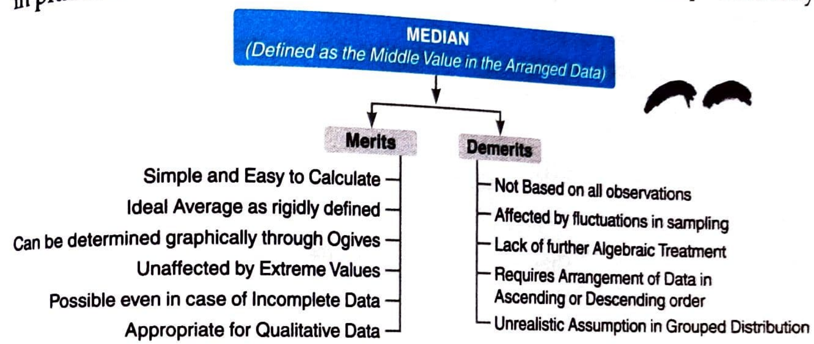
Demerits of Median

1. Not based on all observations: Median, being a positional average, is not based on each and every item of the distribution.

   For example, the median of 10, 25, 50, 60 and 65 is 50. If we replace the observations 10 and 25 by any two values smaller than 50 and replace 60 and 65 with two values greater than 50, then value of the median will remain same.

2. Affected by fluctuations in sampling: It is affected by the fluctuations of sampling. Thus, if class-intervals are not uniform, the value of median becomes inappropriate.
3. Lack of further algebraic treatment: The median is not capable of algebraic treatment. *For example*, median cannot be used for determining combined median of two or more groups as is possible in case of mean.

4. Arrangement required: Since median is an average of position, therefore arranging the data in ascending or descending order of magnitude is time consuming in case of large number of observations.
5. Unrealistic assumption in case of grouped distribution: The formula for the computation of median, in case of grouped frequency distribution, is based on the assumption that the observations in the median class are uniformly distributed. This assumption is rarely met in practice.



9.9 APPLICATIONS OF MEDIAN

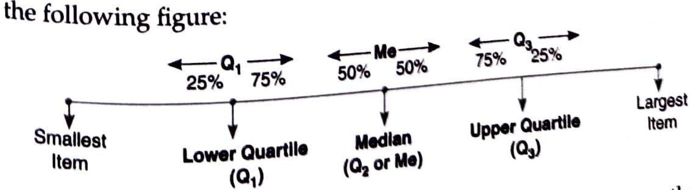
The median is helpful in understanding the characteristic of a data set when:

1. Observations are qualitative in nature;
2. Extreme values are present in the data set;
3. A quick estimate of an average is desired.

9.10 QUARTILES

Median is a value which splits the series in two equal parts. Similarly, there are other positional values, which *divide* a series in a number of parts. The most common positional values besides median are *Quartiles*.

Quartiles divide a series into four equal parts. For any series, there will be three quartiles as shown by the following figure:



1. First or Lower Quartile (Q_1): Q_1 divides the distribution in such a way that one-fourth (25%) of total items fall below it and three-fourth (75%) fall above it.
2. Second Quartile (Q_2) or Median: It has already been discussed.

3. Third or Upper Quartile (Q_3): Q_3 divides the distribution in such a way that three-fourth (75%) of total items fall below it and one-fourth (25%) fall above it.

Percentiles – For Knowledge Enrichment

1. The percentile values divide the distribution into 100 parts each containing 1 per cent of the observations.
2. There are, in all, 99 percentiles denoted as P_1, P_2, ... P_{99} respectively. P_{50} is the median value.
3. If you have secured 60 percentile in an examination, it means that your position is below 40 percent of total candidates appeared in the examination.

9.11 COMPUTATION OF QUARTILES

The computation of quartiles is done exactly in the same manner as the computation of the Median. *While calculating Q_1 and Q_3 the series have to be arranged in ascending or descending order as in case of median.*

Individual Series

In case of individual series, the values of lower quartile (Q_1) and upper quartile (Q_3) would be the size of $\left[\frac{N+1}{4}\right]^{th}$ and 3 $\left[\frac{N+1}{4}\right]^{th}$ item respectively.

Example 24. From the data given below, calculate lower quartile (Q_1) and upper quartile (Q_3):

Pocket money (in ₹)	46	35	28	52	54	43	35	49	46	50	41

Solution:

Calculation of Lower Quartile (Q_1) and Upper Quartile (Q_3)

S. No.	Pocket money (in ₹) arranged in ascending order
1	28
2	35
3	35
4	41
5	43
6	46
7	46
8	49
9	50
10	52
11	54
N = 11	

Calculation of Lower Quartile (Q_1)

Q_1 = Size of $\left[\frac{N+1}{4}\right]^{th}$ item = Size of $\left[\frac{11+1}{4}\right]^{th}$ item = Size of 3rd item

Q_1 = ₹ 35

Calculation of Upper Quartile (Q_3)

Q_3 = Size of 3 $\left[\frac{N+1}{4}\right]^{th}$ item = Size of 3 $\left[\frac{11+1}{4}\right]^{th}$ item = Size of 9th item

Q_3 = ₹ 50

Ans. Lower Quartile (Q_1) = ₹ 35; Upper Quartile (Q_3) = ₹ 50

Example 25. Calculate first quartile and third quartile from following data:

Marks of Students	60	38	46	43	50	58	65	69

Solution:

Arranging marks in ascending order, we get: 38, 43, 46, 50, 58, 60, 65, 69

Calculation of Lower Quartile (Q_1)

Q_1 = Size of $\left[\frac{N+1}{4}\right]^{th}$ item = Size of $\left[\frac{8+1}{4}\right]^{th}$ item = Size of 2.25th item

Size of 2.25th item = Size of 2nd item + .25 times (Size of 3rd item – Size of 2nd item)

Size of 2.25th item = 43 + .25 (46 – 43) = 43 + .75 = 43.75

Q_1 = 43.75 Marks

Calculation of Upper Quartile (Q_3)

Q_3 = Size of 3 $\left[\frac{N+1}{4}\right]^{th}$ item = Size of 3 $\left[\frac{8+1}{4}\right]^{th}$ item = Size of 6.75th item

Size of 6.75th item = Size of 6th item + .75 times (Size of 7th item – Size of 6th item)

Size of 6.75th item = 60 + .75 (65 – 60) = 60 + .75 (5) = 63.75

Q_3 = 63.75 marks

Ans. Lower Quartile (Q_1) = 43.75 marks; Upper Quartile (Q_3) = 63.75 marks

Discrete Series

In case of discrete series also, the values of lower quartile (Q_1) and upper quartile (Q_3) would be the Size of $\left[\frac{N+1}{4}\right]^{th}$ and 3 $\left[\frac{N+1}{4}\right]^{th}$ items respectively. *However, for value of N, the cumulative frequency is calculated.*

The following example will illustrate this.

Example 26. From the following, compute Q_1 and Q_3.

X	10	20	30	40	50	60	70
f	2	3	5	10	5	3	2

Solution:

We first calculate the cumulative frequency:

X	f	c.f.
10	2	2
20	3	5
30	5	10
40	10	20
50	5	25
60	3	28
70	2	30
	N = Σf = 30	

Calculation of Lower Quartile (Q_1)

$$Q_1 = \text{Size of } \left[\frac{N+1}{4}\right]^{th} \text{ item} = \text{Size of } \left[\frac{30+1}{4}\right]^{th} \text{ item} = \text{Size of } 7.75^{th} \text{ item}$$

7.75^{th} item falls in the cumulative frequency of 10 and the size against this cumulative frequency is 30. Therefore, Q_1 is 30.

Calculation of Upper Quartile (Q_3)

$$Q_3 = \text{Size of } 3\left[\frac{N+1}{4}\right]^{th} \text{ item} = \text{Size of } 3\left[\frac{30+1}{4}\right]^{th} \text{ item} = \text{Size of } 23.25^{th} \text{ item}$$

23.25^{th} item falls in the cumulative frequency of 25 and the size against this cumulative frequency is 50. So, Q_3 is 50.

Ans. Lower Quartile (Q_1) = 30; Upper Quartile (Q_3) = 50

Continuous Series

In case of continuous series, the **lower quartile (Q_1)** is the $\left[\frac{N}{4}\right]^{th}$ item and the exact value of Q_1 is calculated by the following formula:

$$Q_1 = l_1 + \frac{\frac{N}{4} - c.f.}{f} \times i$$

*Where, l_1 = Lower limit of the quartile class; **c.f.** = Cumulative frequency of the class preceding quartile class; **f** = Simple frequency of the quartile class; **i** = Class-internal of the quartile class.*

Similarly, the **upper quartile (Q_3)** is the $3\left[\frac{N}{4}\right]^{th}$ item and the exact value of Q_3 is calculated by the following formula:

$$Q_3 = l_1 + \frac{\frac{3N}{4} - c.f.}{f} \times i$$

Let us understand the calculations of Q_1 and Q_3 with the help of following example.

Example 27. With the help of following details, calculate lower quartile and upper quartile.

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of Students	16	14	23	17	7	3

Solution:

Marks (X)	No. of Students (f)	c.f.
0–10	16	16
10–20	14	30
20–30	23	53
30–40	17	70
40–50	7	77
50–60	3	80
	N = Σf = 80	

Calculation of Lower Quartile (Q_1)

$$Q_1 = \frac{N}{4} = \frac{80}{4} = 20^{th} \text{ item}$$

20^{th} item lies in the group 10–20

$l_1 = 10$, c.f. = 16, f = 14, i = 10

By applying formula:

$$Q_1 = l_1 + \frac{\frac{N}{4} - c.f.}{f} \times i = 10 + \frac{20-16}{14} \times 10 = 12.86 \text{ Marks}$$

$Q_1 = 12.86$ marks

Calculation of Upper Quartile (Q_3)

$$Q_3 = \frac{3N}{4} = \frac{240}{4} = 60^{th} \text{ item}$$

60^{th} item lies in the group 30–40

$l_1 = 30$, c.f. = 53, f = 17, i = 10

$$Q_3 = l_1 + \frac{\frac{3N}{4} - c.f.}{f} \times i = 30 + \frac{60-53}{14} \times 10 = 34.12 \text{ marks}$$

$Q_3 = 34.12$ marks

Ans. Lower Quartile (Q_1) = 12.86 marks; Upper Quartile (Q_3) = 34.12 marks

Example 28. Calculate the value of lower quartile, median and upper quartile from the following data:

Class-interval (less then)	10	20	30	40	50
Frequency	22	60	106	141	161

Solution:

In the given example, the data is given in the form of cumulative series. So, it will be first converted into simple series to calculate the median class and quartiles class.

Class-Interval (X)	Frequency (f)	c.f.
0–10	22	22
10–20	38	60
20–30	46	106
30–40	35	141
40–50	20	161
	N = Σf = 161	

Calculation of Lower Quartile (Q_1)

$$Q_1 = \frac{N}{4} = \frac{161}{4} = 40.25^{th} \text{ item}$$

40.25^{th} item lies in the group 10–20

$l_1 = 10$, c.f. = 22, f = 38, i = 10

By applying formula:

$$Q_1 = l_1 + \frac{\frac{N}{4} - c.f.}{f} \times i = 10 + \frac{40.25 - 22}{38} \times 10 = 14.80$$

$Q_1 = 14.80$

Calculation of Median (Me)

$$Me = \frac{N}{2} = \frac{161}{2} = 80.5^{th} \text{ item}$$

80.5^{th} item lies in the group 20–30

$l_1 = 20$, c.f. = 60, f = 46, i = 10

$$Me = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i = 20 + \frac{80.5 - 60}{46} \times 10 = 24.45$$

Median = 24.45

Calculation of Upper Quartile (Q_3)

$$Q_3 = \frac{3N}{4} = \frac{483}{4} = 120.75^{th} \text{ item}$$

120.75^{th} item lies in the group 30–40

$l_1 = 30$, c.f. = 106, f = 35, i = 10

$$Q_3 = l_1 + \frac{\frac{3N}{4} - c.f.}{f} \times i = 30 + \frac{120.75 - 106}{35} \times 10 = 34.21$$

$Q_3 = 34.21$

Ans. Lower Quartile (Q_1) = 14.80; Median = 24.45; Upper Quartile (Q_3) = 34.21

9.12 MODE

Mode is another important measure of central tendency, which is conceptually very useful. ***Mode is the value occurring most frequently in a set of observations and around which other items of the set cluster most densely.*** Mode is the value with the max. frequency

Actually the word 'mode' has been derived from the French word 'La Mode' which signifies the most fashionable values of a distribution, because it is repeated the highest number of times in the series. Thus, *Mode is the value which occur the largest number of times in a series.*

Example: *If the shoe size of 10 people is: 8, 9, 7, 9, 10, 9, 10, 9, 11, 8; mode can be conveniently found by arranging the observations in an ascending order (7, 8, 8, 9, 9, 9, 9, 10, 10, 11) and counting the number of times each observation occurs. Mode size of shoes is 9 as it occur the maximum number of times (four times).*

Definitions of Mode

In the words of A.M. Tuttle, "Mode is the value which has the greatest frequency density in its immediate neighborhood".

In the words of Croxton and Cowden, "Mode of a distribution is the value at the point around which the items tend to be most heavily concentrated".

Important Points about Mode

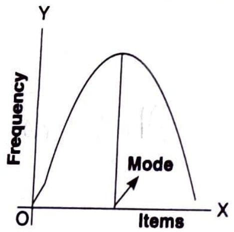
- Mode is extensively used to measure taste and preferences of people for a particular brand of the commodity.
- In case of frequency distribution, mode is determined by the value corresponding to maximum frequency.
- The value of mode is denoted by the symbol 'Z'.
- **Mode is preferable to mean and median** when it is desired to know the most typical value. *For example,* the most common size of shoes, the most common size of a ready-made garment, the most common size of pocket expenditure of a student, the most popular candidate in an election, etc.
- A distribution can either be uni-modal, bi-modal or multi-modal. However, if each observation occurs the same number of times in a series, then there is no mode in that distribution.
  - (i) *No Modal Value:* When each observation occur the same number of times in a series;
  - (ii) *Uni-modal:* When one item occur the maximum number of times;

(iii) *Bi-modal:* When two items have the same maximum frequency;

(iv) *Multi-modal:* When more than two items have the same maximum frequency.

Mode with Frequency Curve

If the nature of mode is to be explained graphically, it is obvious that the mode would be the point of maximum frequency which is indicated by the peak of a frequency curve.



In the given diagram, X-axis denotes the value of variable and Y-axis the corresponding frequencies. *Mode is that value on the X-axis,* which correspond to the maximum frequency on the Y-axis.

9.13 CALCULATION OF MODE

The value of mode can be calculated in the following series:

1. ~~Individual Series~~
2. ~~Discrete Series~~
3. ~~Continuous Series~~

Grouping

Individual Series

There are two methods of finding out mode in an individual series:

1. By Observation;
2. By Converting individual series into a Discrete Series, *i.e,* by frequency distribution.

Mode by Way of Observation

Through observation, one can notice the occurrence of items in a distribution.

Step 1. Arrange the data in ascending or descending order.

Step 2. The item which occurs most in the series is *'Mode'.*

Example 29. From the heights of 15 students, calculate the value of mode.

Height (in inches)	52	50	66	70	66	72	71	66	60	67	69	67	48	60	65

Solution:

By arranging the series in an ascending order, we get:

48	50	52	60	60	65	66	66	67	67	69	70	71	72

By observation, height 66 inches occurs most, therefore, the mode (Z) is 66 inches.

Mode by Converting Individual Series into Discrete Series

If number of items in an individual series are more, then the individual series can be converted into discrete series. *Mode is then calculated as the value corresponding to the highest frequency.*

Example 30. Calculate the value of Mode from the data given in Example 29 by converting the data into discrete series:

Solution:

Heights (in inches)	*Frequency*
48	1
50	1
52	1
60	2
65	1
66	**3**
67	2
69	1
70	1
71	1
72	1
Total	**15**

The height of 66 inches has the maximum frequency. Therefore, mode height, i.e. (Z) is 66.

Ans. Mode = 66 inches

Example 31. Find out the mode from the followings figures by: (i) Observation Method; (ii) Frequency distribution Method.

57	50	60	65	80	40	43	63	70	60	53	57	63	53	57	60	57

Solution:

(I) Observation Method

By arranging the series in an ascending order, we get:

40	43	50	53	53	57	57	57	57	60	60	60	63	63	65	70	80

By observation, 57 occurs most, therefore, the mode (Z) is 57.

(II) Frequency Distribution Method

In order to find out mode we have to convert the individual series into discrete series.

Items	*Frequency*
40	1
43	1
50	1
53	2
57	4

60		3
63		2
65		1
70		1
80		1
Total		**17**

Item 57 occurs the largest number of times. So, mode (Z) = 57.

Ans. Mode = 57

Discrete Series

There are two methods to determine mode in a discrete series:

(i) Mode by Observation, known as Inspection Method

(ii) Mode by Grouping Method.

Let us discuss these two methods in detail:

(i) Mode by Observation

The mode can be determined by inspection if:

- Frequencies are regular and homogeneous; and
- There is only one item which has the maximum frequency.

In such a case, the value corresponding to the highest frequency would be the modal value.

This is illustrated in the Example 32.

Mode

Example 32. Find out mode of the following series.

Daily Wages (in ₹)	100	110	120	130	140	150
No. of persons	2	4	8	10	5	4

Solution.

By inspection, we can see that 130 occurs most frequently in the series, hence modal daily wages = ₹ 130.

(ii) Mode by Grouping Method

If the frequency distribution is irregular and heterogeneous, then it is not necessary that mode is always the value which occurs most frequently or whose frequency is the maximum. In such cases, *Grouping Method* is generally used for obtaining the mode.

According to grouping method, 2 tables are prepared to determine the modal value:

1. Grouping Table: In this first table, groupings of frequencies are presented in six columns.
2. Analysis Table: In this second table, occurrence of frequencies or values in various groupings are written and added. *Modal value is the value which occurs in the maximum number of groupings.*

Steps of Grouping Method

Prepare a table consisting of 6 columns, in addition to a column for various values of X.

Column 1: Write the frequencies against various values of X, as given in the question;

Column 2: Group frequencies in two's starting from the top. Find out their total and mark the highest total;

Column 3: Group frequencies in two's starting from the second frequency (i.e. first frequency is left out). Find out their total and mark the highest total;

Column 4: Group frequencies in three's starting from the top. Find out their total and mark the highest total;

Column 5: Group frequencies in three's starting from the second frequency (i.e. first frequency is left out). Find out their total and mark the highest total;

Column 6: Group frequencies in three's starting from the third frequency (i.e. first and second frequencies are left out). Find out their total and mark the highest total.

The highest frequency total in each of the six columns is identified and analysed in the *Analysis Table*, to determine mode.

Example 33. Calculate the value of Mode from the data given in Example 32 by grouping method.

Solution:

First of all, grouping of the data is done.

Grouping Table

Wages in ₹ (X)	No. of Persons (f)	In Two's		In Three's		
	Column I	Column II	Column III	Column IV	Column V	Column VI
100	2	2 + 4 = 6		2 + 4 + 8 = 14		
110	4		4 + 8 = 12		4 + 8 + 10 = 22	
120	8	8 + 10 = **18**				8 + 10 + 5 = 23
130	**10**		10 + 5 = 15	10 + 5 + 4 = 19		
140	5	5 + 4 = 9				
150	4					

After having prepared Grouping Table, we are required to prepare an Analysis Table. In this table, we enter the values having maximum frequencies in each column of Grouping Table by mean of ticks (✓) as follows:

Analysis Table

Column No.	100	110	120	130	140	150
I				✓		
II			✓	✓		
III				✓	✓	

IV				✓	✓	✓
V		✓	✓	✓		
VI			✓	✓	✓	
Total		**1**	**3**	**6**	**3**	**1**

Since the value 130 has occurred the maximum number of times i.e. 6, the **modal income is** ₹ 130.

Ans. Mode = ₹ 130

Example 34. Find out mode of the following series.

Size	8	9	10	11	12	13	14	15
Frequency	5	6	8	7	9	8	9	6

Solution.

The frequencies of two items: 12 and 14 have the highest frequency of 9. So, grouping of frequencies is essential. The method of grouping will be used for determination of mode.

Grouping Table

Size (X)	*Frequency (f)*	*In Two's*		*In Three's*		
	Column I	*Column II*	*Column III*	*Column IV*	*Column V*	*Column VI*
8	5	11		19		
9	6		14		21	
10	8	15				24
11	7		16	24		
12	9	17			26	
13	8		17			23
14	9	15				
15	6					

Analysis Table

Column No.	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*
I					✓		✓	
II					✓	✓		
III						✓	✓	
IV				✓	✓	✓		
V					✓	✓	✓	
VI			✓	✓	✓			
Total	—	—	1	2	5	4	3	—

The size 12 is occurring maximum number of times (5 times). So, Mode = 12.

Ans. Mode = 12

Continuous Series

In continuous series, mode lies in a particular class or group, which is called the *modal class.* The following two methods are used in determining mode:

(i) Observation Method or Inspection Method

(ii) Grouping Method.

Observation Method

If the frequencies are regular, homogeneous and there is a single maximum frequency, then we can use the observation method to determine Mode.

Steps of Observation Method

Step 1. Determine the modal class, i.e. class with the highest frequency;

Step 2. Determine the exact value of mode by the following formula:

$$Mo = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Where,

Mo = Mode

l_1 = Lower limit of modal class

f_1 = Frequency of the modal class

f_0 = Frequency of class preceding the modal class

f_2 = Frequency of class succeeding the modal class

i = Class-interval of the modal class

The formula for calculation of Mode can also be expressed as:

$$Mo = l_1 + \frac{f_1 - f_0}{(f_1 - f_0) - (f_1 - f_2)} \times i$$

Do Consider it

When frequency of pre modal class or post modal class is higher than that of modal class, i.e. if $(2f_1 - f_0 - f_2)$ comes out to be zero or $(f_1 - f_2)$ is negative, then value of mode is obtained by the following formula:

$$Mo = l_1 + \frac{|f_1 - f_0|}{|f_1 - f_0| + |f_1 - f_0|} \times i$$

The symbols have usual meanings (as discussed before). The only difference is that absolute values (ignoring signs) of difference between f_1 and f_0 and between f_1 and f_2 will be taken. **(Refer Examples 40 and 41)**

Example 35. Find out mode of the following series.

Class-Interval	0–5	5–10	10–15	15–20	20–25
Frequency	2	4	15	6	7

Solution:

By inspection, it is clear that modal class is 10–15, because frequency of this class is maximum i.e. 15.

Computation of Mode

Class-Interval	Frequency
0–5	2
5–10	4 f_0
(l_1) 10–15	**15 f_1 Modal Class**
15–20	6 f_2
20–25	7

To calculate mode, the following formula will be used

$$\text{Mode (Z)} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$l_1 = 10, f_1 = 15, f_0 = 4, f_2 = 6, i = 5$

$$Z = 10 + \frac{15-4}{2 \times 15 - 4 - 6} \times 5 = 10 + \frac{11}{20} \times 5 = 12.75$$

Ans. Mode = 12.75

Grouping Method

As discussed before, Inspection Method is of use only when there is regularity and homogeneity in the series. In case of any irregularity, *Grouping Method* is preferred.

Steps of Grouping Method

The determination of mode by grouping method involves two steps:

Step 1. Determine the Modal Class by the process of grouping. *The grouping procedure is same as done under discrete series.*

Step 2. Determine the exact value of mode by the following formula:

$$Mo = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Let us understand the calculation of mode by Grouping Method (under continuous series) with the help of following example.

Example 36. From the following data, determine mode.

Size	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90
Frequency	4	10	25	15	23	22	12	3

Solution:

By inspection, the modal class is not clear. Although 30 – 40 class has the highest frequency (25), yet greatest concentration of items is around 50–60 class (with frequency of 23). Hence, we prepare a Grouping Table and Analysis Table.

Grouping Table

Size (X)	Frequency (f) Column I	In Two's Column II	Column III	In Three's Column IV	Column V	Column VI
10–20	4	14				
20–30	10		35	39		
30–40	25	40			50	
40–50	15		38			63
50–60	23	45		60		
60–70	22		34		57	37
70–80	12	15				
80–90	3					

Analysis Table

Column No.	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90
I			✓					
II					✓	✓		
III				✓	✓			
IV				✓	✓	✓		
V					✓	✓	✓	
VI			✓	✓	✓			
Total	—	—	**2**	**3**	**5**	**3**	**1**	—

It is clear that modal class is 50–60 and frequency of this class is 23.

Using formula:

$$\text{Mode (Z)} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$l_1 = 50, f_1 = 23, f_0 = 15, f_2 = 22, i = 10$

$$Z = 50 + \frac{23-15}{2 \times 23 - 15 - 22} \times 10 = 50 + \frac{8}{9} \times 10 = 58.89$$

Ans. Mode = 58.89

9.14 MODE IN SPECIAL CASES

The calculation process of Mode is different under some special circumstances. Let us discuss these special cases:

MODE IN SPECIAL CASES

Cumulative Frequency Distribution *(Less Than or More Than)*	Mid-Values are Given *(Mid-points of class-intervals are given)*	Inclusive Class-Intervals *(Classes of type 10–19, 20–29 are given)*	Open-End Series *(Lower limit of 1st class and upper limit of last class not given)*	Unequal Class-Intervals *(When class-intervals are not equal)*
Convert it into simple frequency distribution *Refer Examples 37 and 38*	**Convert mid-values into class-intervals** *Refer Example 39*	**Convert class-intervals into Exclusive series** *Refer Example 40*	**No need to find missing limits. Calculate Mode in usual manner** *Refer Example 41*	**Class-intervals have to be made equal and frequencies have to be adjusted** *Refer Example 42*

Cumulative Series ('Less than' or 'More than')

When cumulative frequency distribution ('Less than' or 'More than' type) is given, then the cumulative frequency distribution has to be converted into a simple frequency distribution.

The calculation of mode in cumulative series will be clear from Example 37 ('less than' series) and Example 38 ('more than' series).

Example 37. Find out the mode in the following series:

Size (below)	5	10	15	20	25	30	35
Frequency	1	3	13	17	27	36	38

Solution:

Here, we are given the data in the form of less than cumulative frequency distribution. To compute mode, we shall first arrange the data in the form of frequency distribution with continuous classes.

Calculation of Frequency Table

Size	*Frequency*	*c.f.*
0–5	1	1
5–10	2	3
10–15	10	13
15–20	4	17
20–25	10	27
25–30	9	36
30–35	2	38

In the given series, the distribution is irregular. Also the maximum frequency (10) is repeated. Therefore, we will find mode by the method of grouping.

Grouping Table

Size (X)	*Frequency (f)* Column I	*In Two's* Column II	Column III	*In Three's* Column IV	Column V	Column VI
0–5	1	3		13		
5–10	2		12		16	
10–15	10	14				24
15–20	4		14	23		
20–25	10	19			21	
25–30	9		11			
30–35	2					

Analysis Table

Column No.	0–5	5–10	10–15	15–20	20–25	25–30	30–35
I			✓		✓		
II					✓	✓	
III				✓	✓		
IV				✓	✓	✓	
V					✓	✓	✓
VI			✓	✓	✓		
Total	—	—	2	3	6	3	1

Since the class 20–25 is repeated maximum number (6) of times, it is the modal class.

So, applying the formula:

$$\text{Mode }(Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$l_1 = 20,\ f_1 = 10,\ f_0 = 4,\ f_2 = 9,\ i = 5$

$$Z = 20 + \frac{10-4}{2 \times 10 - 4 - 9} \times 5 = 20 + \frac{6}{7} \times 5 = 24.28$$

Ans. Mode = 24.28

Example 38. Calculate mode from the following particulars:

Daily Wages in ₹ (More than)	100	200	300	400	500
No. of workers	53	48	36	17	6

Solution:

Here, we are given the data in the form of more than cumulative frequency distribution. To compute mode, we shall first arrange the data in the form of frequency distribution with continuous classes.

Calculation of Frequency Table

Daily Wage (₹)	*Frequency*
100–200	5
200–300	12 f_0
(l_1) 300–400	19 (f_1) **Modal Class**
400–500	11 f_2
500–600	6

By inspection, it is clear that modal class is 300–400, because frequency of this class is maximum i.e. 19.
To calculate mode, the following formula will be used

$$\text{Mode (Z)} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$l_1 = 300, f_1 = 19, f_0 = 12, f_2 = 11, i = 100$

$$Z = 300 + \frac{19 - 12}{2 \times 19 - 12 - 11} \times 100 = 300 + \frac{7}{15} \times 100 = 346.67$$

Ans. Mode = ₹ 346.67

Mid-Values are given

In this case, we have to first convert the mid-values in to class-interval to calculate the value of mode.

Example 39. Calculate the mode from the following data:

Marks (Mid-values)	5	15	25	35	45	55	65	75
No. of Students	15	20	25	24	12	31	71	52

Solution:

In the given example, we are given the mid-values. We need to first convert it into continuous series.

Step 1: The difference between the two mid-values is 10.

Step 2: Half of the difference is: $\frac{10}{2} = 5$. Now, 5 is reduced and added to each mid-value to determine the lower limit and upper limit. It is shown in the following table:

Calculation of Class-Intervals

Marks (X)	*No. of Students (f)*
0–10	15
10–20	20
20–30	25
30–40	24
40–50	12
50–60	31
60–70	71
70–80	52

In the given series, the distribution is irregular. Therefore, we will find mode by the method of grouping.

Grouping Table

Size (X)	*Frequency (f)*	*In Two's*		*In Three's*		
	Column I	*Column II*	*Column III*	*Column IV*	*Column V*	*Column VI*
0–10	15	35		60		
10–20	20		45		69	
20–30	25	49				61
30–40	24		36	67		
40–50	12	43			114	
50–60	31		102			154
60–70	71	123				
70–80	52					

Analysis Table

Column No.	*0–10*	*10–20*	*20–30*	*30–40*	*40–50*	*50–60*	*60–70*	*70–80*
I							✓	
II							✓	✓
III						✓	✓	
IV				✓	✓	✓		
V					✓	✓	✓	
VI						✓	✓	✓
Total	—	—	—	**1**	**2**	**4**	**5**	**2**

Since the class 60–70 is repeated maximum number of times, it is the modal class.

So, applying the formula:

$$\text{Mode (Z)} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$l_1 = 60, f_1 = 71, f_0 = 31, f_2 = 52, i = 10$

$$Z = 60 + \frac{71 - 31}{2 \times 71 - 31 - 52} \times 10 = 60 + \frac{40}{59} \times 10 = 66.78$$

Ans. Mode = 66.78 marks

Inclusive Class-Intervals

The frequency distribution must be continuous with exclusive type classes, without any gaps. In case data is not in the form of continuous classes, it must be converted into continuous classes before applying the formula. **Therefore, in case of inclusive class-intervals, the formula remains the same, but the class-intervals are converted into an exclusive class-interval series.**

Example 40. Calculate mode in the following distribution.

Marks	40–49	50–59	60–69	70–79	80–89	90–99
No. of Students	12	30	24	20	12	2

Solution:

In the given example, inclusive class-intervals will be first converted to exclusive class-intervals and, thereafter, mode will be determined.

Calculation of Exclusive Class-Intervals

Marks	No. of Students
39.5–49.5	12
49.5–59.5	30
59.5–69.5	24
69.5–79.5	20
79.5–89.5	12
89.5–99.5	2

By inspection, the modal class is not clear. Although 49.5–59.5 class has the highest frequency of 30, yet greatest concentration of items is around 59.5–69.5 class (with frequency of 24). Therefore, we will find mode by the method of grouping.

Grouping Table

Size (X)	No. of Students (f)	In Two's		In Three's		
	Column I	Column II	Column III	Column IV	Column V	Column VI
39.5–49.5	12	42		66		
49.5–59.5	30		54		74	
59.5–69.5	24	44				56
69.5–79.5	20		32	34		
79.5–89.5	12	14				
89.5–99.5	2					

Analysis Table

Column No.	39.5–49.5	49.5–59.5	59.5–69.5	69.5–79.5	79.5–89.5	89.5–99.5
I		✓				
II			✓	✓		
III		✓	✓			
IV	✓	✓	✓			
V		✓	✓	✓		
VI			✓	✓	✓	
Total	1	4	5	3	1	—

From the analysis table, the modal group is 59.5–69.5. The frequency of this group is 24. By applying the formula:

$$\text{Mode } (Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

But in the given example, f_1 (24) is less than f_0 (30). It means, $(f_1 - f_0)$ will be negative. In such cases, mode is calculated by the following formula:

$$\text{Mode } (Z) = l_1 + \frac{|f_1 - f_0|}{|f_1 - f_0| + |f_1 - f_2|} \times i$$

$l_1 = 59.5,\ f_1 = 24,\ f_0 = 30,\ f_2 = 20,\ i = 10$

$$Z = 59.5 + \frac{|24 - 30|}{|24-30| + |24-20|} \times 10 = 59.5 + \frac{6}{10} \times 10 = 65.5$$

Ans. Mode = 65.5 Marks

Open-End Series

In case of open-end classes, the lower limit of the first class and upper limit of the last class is not given. ***To calculate Mode, there is no need to complete the class-interval.***

Example 41 would illustrate the point.

Example 41. Calculate the value of mode from the following particulars.

Class-Intervals (X)	Frequency (f)
Below 20	4
20–30	6
30–40	5
40–50	10
50–60	20
60–70	22
70–80	24
80–90	6
90–100	2
Above 100	1

Solution:

The given data consist of open-end classes. However, to calculate mode, there is no need to complete the class-interval.

By inspection, the modal class is not clear. Although 70–80 class has the highest frequency (24), yet greatest concentration of items is around 60–70 class (with frequency of 22). *Hence, we prepare a Grouping Table* **and Analysis Table.**

Grouping Table

Class-Interval (X)	Frequency (f) Column I	In Two's Column II	In Two's Column III	In Three's Column IV	In Three's Column V	In Three's Column VI
Below 20	4	10		15		
20–30	6		11		21	
30–40	5	15				35
40–50	10		30	52		
50–60	20	**42**				
60–70	22		**46**		**66**	52
70–80	**24**	30		32		
80–90	6		8		9	
90–100	2	3				
Above 100	1					

Analysis Table

Column No.	Below 20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100	Above 100
I							✓			
II					✓	✓				
III						✓	✓			
IV				✓	✓	✓				
V					✓	✓	✓			
VI						✓	✓	✓		
Total	—	—	—	1	3	5	4	1	—	—

It is clear that modal class is 60–70 and frequency of this class is 22.

Using formula:

$$\text{Mode (Z)} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$l_1 = 60, f_1 = 22, f_0 = 20, f_2 = 24, i = 10$

$$Z = 60 + \frac{22 - 20}{2 \times 22 - 20 - 24} \times 10$$

However, the value of $(2f_1 - f_0 - f_2)$ is zero. In such cases, mode is calculated by the following formula:

$$\text{Mode (Z)} = l_1 + \frac{|f_1 - f_0|}{|f_1 - f_0| + |f_1 - f_2|} \times i$$

$$Z = 60 + \frac{|22-20|}{|22-20| + |22-24|} \times 10 = 60 + \frac{2}{4} \times 10 = 65$$

Ans. Mode = 65

Unequal Class-Intervals

Mode can be calculated only if the class-intervals are of equal magnitude. If unequal class-intervals are given, then we must make them equal before we calculate mode. **The class-intervals should be made equal and frequencies be adjusted.** It is assumed that frequencies are equally distributed.

The following example will illustrate the point.

Example 42. Find the mode from the following data:

Class-interval	0–10	10–20	20–40	40–50	50–70	70–80
Frequency	10	14	40	35	42	10

Solution:

In the given example, the class-intervals are not equal. To calculate mode, the class-intervals are made equal and frequencies are adjusted. We take the assumption that in this case, frequencies are equally distributed.

Calculation of Frequency Table

Class-Interval	Frequency
0–10	10
10–20	14
20–30	20
30–40	20 (f_0)
(l_1) 40–50	35 (f_1) **Modal Class**
50–60	21 (f_2)
60–70	21
70–80	10

By inspection, it is clear that modal class is 40–50 as frequency of this class is maximum i.e. 35.

To calculate mode, the following formula will be used:

$$\text{Mode (Z)} = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

$l_1 = 40, f_1 = 35, f_0 = 20, f_2 = 21, i = 10$

$$Z = 40 + \frac{35 - 20}{2 \times 35 - 20 - 21} \times 10 = 40 + \frac{15}{29} \times 10 = 45.17$$

Ans. Mode = 45.17

SUMMARY OF MODE IN SPECIAL CASES

CASE 1: Cumulative Frequency Distribution *(Less Than Series)*: Convert it into Simple Frequency Distribution and then calculate Mode in usual manner.

Marks	Less than 10	Less than 20	Less than 30	Less than 40	Less than 50
Students	2	6	21	27	34

Marks (X)	Students (f)
0 – 10	2
10 – 20	4
20 – 30	15
30 – 40	6
40 – 50	7

By inspection, it is clear that modal class is 20–30.

$l_1 = 20$ $f_1 = 15$ $f_0 = 4$ $f_2 = 6$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 20 + \frac{15 - 4}{2 \times 15 - 4 - 6} \times 10 = 12.75$$

CASE 2: Mid-Values are Given: When Mid-points are given, then convert such mid-values into regular Class-Intervals and then calculate Median in usual manner.

Mid-Points	5	15	25	35	45
Frequency	8	20	40	10	18

Class-Intervals (X)	Frequency (f)
0 – 10	8
10 – 20	20
20 – 30	40
30 – 40	10
40 – 50	18

By inspection, it is clear that modal class is 20–30.

$l_1 = 20$ $f_1 = 40$ $f_0 = 20$ $f_2 = 10$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 20 + \frac{40 - 20}{2 \times 40 - 20 - 10} \times 10 = 24$$

CASE 3: Inclusive Class-Intervals *(Classes of type 10-19, 20-29 are given)*: Convert Inclusive Class-Intervals into Exclusive Series.

Class-Intervals	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
Frequency	9	10	22	40	18

Class-Intervals (X)	Frequency (f)
9.5 – 19.5	9
19.5 – 29.5	10
29.5 – 39.5	22
39.5 – 49.5	40
49.5 – 59.5	18

By inspection, it is clear that modal class is 39.5–49.5 as frequency of this class is maximum, i.e. 40.

$l_1 = 39.5$ $f_1 = 40$ $f_0 = 22$ $f_2 = 18$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 39.5 + \frac{40 - 22}{2 \times 40 - 22 - 18} \times 10 = 44$$

CASE 4: Open-End Series *(Lower limit of first class and upper limit of last class not given)*: There is no need to find missing limits, i.e. calculate Mode in usual manner.

Class-Intervals	Less than 40	40 – 50	50 – 60	60 – 70	More than 70
Frequency	3	14	24	9	5

Class-Intervals (X)	Frequency (f)
Less than 40	3
40 – 50	14
50 – 60	24
60 – 70	9
More than 70	5

By inspection, it is clear that modal class is 50–60.

$l_1 = 50$ $f_1 = 24$ $f_0 = 14$ $f_2 = 9$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 50 + \frac{24 - 14}{2 \times 24 - 14 - 9} \times 10 = 54$$

CASE 5a: Unequal Class-Intervals *(When Class-Intervals are Merged together)*: Before calculating mode, class-intervals are made equal and frequencies are adjusted.

X	0 – 5	5 – 10	10 – 15	15 – 20	20 – 30	30 – 40
f	4	6	7	5	30	8

To make the class-intervals equal, 0-5 and 5-10 are merged together to make the class-interval of 0-10 with frequency of 10 (= 4 + 6). Similarly, 10-15 and 15-20 are merged together to make the class-interval of 10-20 with frequency of 12 (= 7 + 5).

Class-Intervals (X)	Frequency (f)
0 – 10	10
10 – 20	12
20 – 30	30
30 – 40	8

By inspection, it is clear that modal class is 20–30.

$l_1 = 20$ $f_1 = 30$ $f_0 = 12$ $f_2 = 8$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 20 + \frac{30 - 12}{2 \times 30 - 12 - 8} \times 10 = 24.5$$

CASE 5b: Unequal Class-Intervals *(When Class-Intervals are Split-up)*: Before calculating mode, class-intervals are made equal and frequencies are adjusted.

X	10 – 20	20 – 40	40 – 50	50 – 70
f	9	32	36	12

To make the class-intervals equal, 20-40 is split up as 20-30 and 30-40 with frequency of 16 (= 32 ÷ 2) each. Similarly, 50-70 is split up as 50-60 and 60-70 with frequency of 6 (= 12 ÷ 2) each.

Class-Intervals (X)	Frequency (f)
10 – 20	9
20 – 30	16
30 – 40	16
40 – 50	36
50 – 60	6
60 – 70	6

By inspection, it is clear that modal class is 40-50.

$l_1 = 40$ $f_1 = 36$ $f_0 = 16$ $f_2 = 6$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 40 + \frac{36 - 16}{2 \times 36 - 16 - 6} \times 10 = 44$$

9.15 MODE BY GRAPHICAL METHOD

Mode can be located graphically with the help of histogram.

Steps to Determine Mode by Graphical Method

Step 1. Draw a histogram of the given data.

Step 2. The rectangle with the greatest height will be the modal class.

Step 3. Draw a line joining the top right point of the rectangle of the modal class with the top right point of the rectangle of the class preceding the modal class.

Step 4. Similarly, draw a line joining the top left point of the rectangle of the modal class with the top left point of the rectangle of the class succeeding the modal class.

Step 5. From the point of intersection of two diagonal lines, draw a perpendicular on the X-axis.

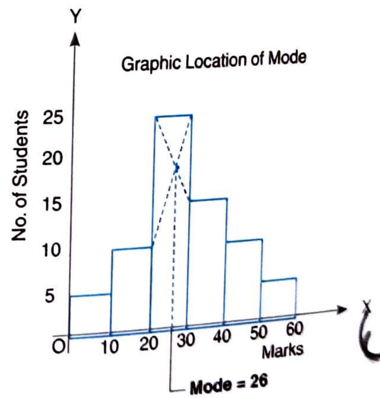
Step 6. The point at which the perpendicular touches the X-axis gives the modal value.

The Graphical Method will be clear from the following example:

Example 43. Find out the mode of the following series, using the Graphic Method.

Marks	0–10	10–20	20–30	30–40	40–50	50–60
No. of Students	5	10	25	15	10	5

Solution:



Verification: By Inspection, we find that modal class is 20–30. Applying the formula:

$$\text{Mode } (Z) = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$$

Given: $l_1 = 20$, $f_1 = 25$, $f_0 = 10$, $f_2 = 15$, $i = 10$

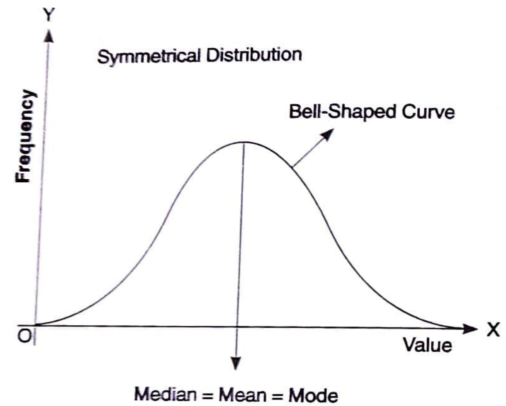
$$Z = 20 + \frac{25 - 10}{2 \times 25 - 10 - 15} \times 10 = 20 + \frac{150}{25} = 26$$

Ans. Mode = 26 Marks

9.16 RELATIONSHIP BETWEEN MEAN, MEDIAN AND MODE

The relationship between mean, median and mode depends upon the nature of distribution, which may be either symmetrical or asymmetrical.

1. Symmetrical Distribution: In case of symmetrical distribution, the values of mean, median and mode are equal, i.e. for symmetrical curves, Mean (X) = Median (Me) = Mode (Z). The symmetrical distribution gives the shape of bell as seen in following figure:

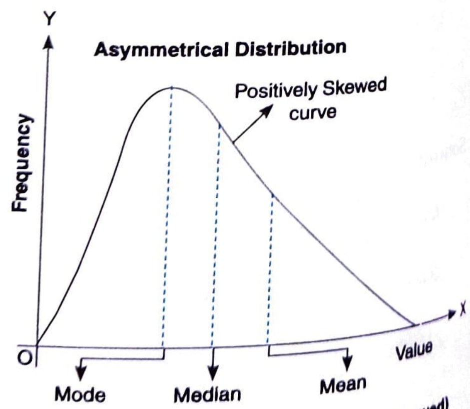


Symmetrical Distribution

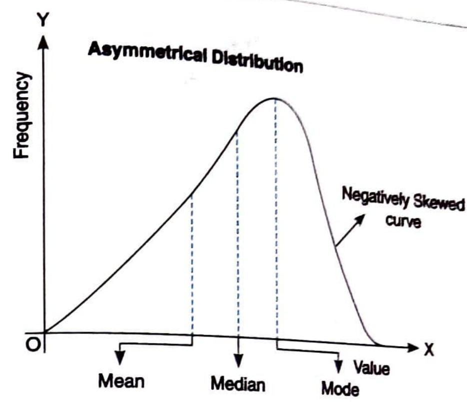
Mode touches the peak of the curve indicating maximum frequency; Median divides the area of the curve in two equal halves and Mean is the centre of gravity.

2. Asymmetrical Distribution: In actual life, most of the distributions are not symmetrical. In an asymmetrical series, mean, median and mode have different values. The frequency curve is not bell shaped, i.e. height of the curve is not in the middle. *An asymmetrical (skewed) distribution is either positively skewed or negatively skewed.*

- For a positively skewed distribution, most of the values of observations in a distribution fall to the right of mode. The order of magnitude of these measures will be: *Mean > Median > Mode*
- For a negatively skewed distribution, values of lower magnitude are concentrated more to the left of the mode. The order of magnitude of these measures will be: *Mean < Median < Mode.*



Asymmetrical Distribution (Positively Skewed)



Asymmetrical Distribution (Negatively Skewed)

Relationship between Mean, Median and Mode in an Asymmetrical Distribution

According to Karl Pearson, the relationship between mean, median and mode in an asymmetrical distribution is given by:

Mode = 3 Median – 2 Mean

1. This formula is specially useful to determine the value of mode, when it is ill-defined.
2. If we know any two of the three values (mean, median and mode), the third can be estimated by using the given formula. The value so computed will be more or less same as obtained by using exact formula, provided distribution is moderately asymmetrical. *(Refer Examples 44, 45, 46, 47 and 48)*

Mode = 3(median) – 2(mean)

Example 44. If the mean and median of moderately asymmetrical series are 26.8 and 27.9 respectively. Calculate the value of mode.

Solution:

Using the empirical relationship, we know:

Mode = 3 Median – 2 Mean = (3 × 27.9) – (2 × 26.8) = 83.7 – 53.6 = 30.1

Ans. Mode = 30.1

Example 45. If mean of a series is 30 and mode is 25. Find Median.

Solution:

Using the empirical relationship, we know:

Mode = 3 Median – 2 Mean

25 = 3 Median – (2 × 30)

3 Median = 25 + 60

$$\text{Median} = \frac{85}{3} = 28.33$$

Ans. Median = 28.33

4. Indeterminate: The value of mode may not always be determined. It is difficult to locate modal class in the case of bi-modal and multi-modal distributions.

5. Affected by the fluctuations of sampling: As compared to mean, mode is affected to a great extent, by sampling fluctuations.

MODE
(Value occurring most frequently in a set of observations)

Merits
- Easy and simple to calculate
- Not affected by Extreme Items
- Can be determined in open-end distributions without ascertaining class limits
- Can be used to describe Quantitative as well as Qualitative Data
- Can be determined graphically through histogram
- Best representative of the data

Demerits
- Not rigidly defined
- Not based on all the observations of a series
- Not capable of Algebraic Treatment
- Indeterminate Value
- Affected by fluctuations of sampling

9.18 COMPARISON BETWEEN MEAN, MEDIAN AND MODE

We have discussed the concepts of mean, median and mode in detail. However, the choice of which method to use, for a given set of data, depends upon number of considerations *(Discussed in Chapter 8, Section 8.4)*, which can be classified into the following broad heads:

1. Rigidly defined: Mean and median are rigidly defined, whereas mode is not rigidly defined in all the situations.
2. Based on all observations: An appropriate average should be based on all the observations. This characteristic is met only by mean and not by median or mode.
3. Possess sampling stability: The preference should be given to mean when the requirement of least sampling variations is to be fulfilled.
4. Further algebraic treatment: It should be capable of further mathematical treatment. This characteristic is satisfied only by mean and, consequently, most of the statistical theories use mean as a measure of central tendency.
5. Easy to understand and calculate: An average should be easy to understand and easy to interpret. This characteristic is satisfied by all the three averages.
6. Not affected by extreme values: It should not be unduly affected by the extreme observations. The mode is most suitable average from this point of view. Median is only slightly affected while mean is very much affected by the presence of extreme observations.

Conclusion: *Generally, arithmetic mean is regarded as the best measure of central tendency and is most widely used in practice. However, in some specific cases, mode or median are also used, depending upon the nature of available data.*

9.19 CALCULATION OF MEAN, MEDIAN AND MODE IN SPECIAL CASES

The calculation process of Mean, Median and Mode is different under some circumstances. Let us have a quick recap of treatment of special cases:

Cases	MEAN	MEDIAN	MODE	Example
Cumulative Series *('Less than' or 'More than')*	Convert the cumulative frequency into a simple frequency distribution and then calculate mean in the usual manner.	Convert the cumulative frequency into a simple frequency distribution in order to find out the frequency of median class and then calculate median in the usual manner.	Convert the cumulative frequency into a simple frequency distribution and then calculate mode in the usual manner.	49, 50
Mid-Values are given	Calculate mean in usual manner. Do not convert mid-values into class-intervals.	Convert the mid-values into Class-intervals and then calculate median.	Convert the mid-values into Class-intervals to calculate mode.	51
Inclusive Class-Intervals	Calculate mean in usual manner. Do not convert the series into an exclusive class-interval series.	Class-intervals are converted into an exclusive class-interval series to calculate median.	Class-intervals are converted into an exclusive class-interval series and, thereafter, mode is calculated.	52
Open-End Series	To calculate mean, missing class limits are assumed, which depends on the pattern of class-intervals of other classes.	Median is calculated in the usual manner without completing the class-intervals.	Mode is calculated in the usual manner without completing the class-intervals.	53
Unequal Class-Intervals	Mean can be determined in the usual manner after calculating the mid-values of each interval. Class-intervals are not made equal.	In case of median also, class-intervals are not made equal and median is calculated in the usual manner.	To calculate mode, it is essential to make class-intervals equal and frequencies have to be adjusted.	54

SUMMARY OF MEAN, MEDIAN AND MODE IN SPECIAL CASES

Cumulative Frequency Distribution
(Less Than Series)

Example 49. Calculate Mean, Median and Mode:

Age in Years (Less than)	10	20	30	40	50	60
No. of Persons	15	32	51	78	97	110

MEAN: Convert it into Simple Frequency Distribution and then calculate Mean in usual manner.

Age in years (X)	No. of Persons (f)	Mid-value (m)	$d = m - A$ (A = 25)	$d' = \frac{m-A}{C}$ (C = 10)	fd'
0 – 10	15	5	–20	–2	–30
10 – 20	17	15	–10	–1	–17
20 – 30	19	25	0	0	0
30 – 40	27	35	10	1	27
40 – 50	19	45	20	2	38
50 – 60	13	55	30	3	39
	$\Sigma f = 110$				$\Sigma fd' = 57$

$$\text{Mean } (\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 25 + \frac{57}{110} \times 10 = 30.18 \text{ years}$$

MEDIAN: Convert it into Simple Frequency Distribution and then calculate Median in usual manner.

Age in years (X)	No. of Persons (f)	c.f.
0 – 10	15	15
10 – 20	17	32
20 – 30	19	51
30 – 40	27	78
40 – 50	19	97
50 – 60	13	110
	$N = \Sigma f = 110$	

$Me = \frac{N}{2} = \frac{110}{2} = 55^{th}$ item; 55^{th} item lies in group 30 – 40

$l_1 = 30$ c.f. = 51 f = 27 i = 10

$$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 30 + \frac{55 - 51}{27} \times 10 = 31.48 \text{ years}$$

MODE: Convert it into Simple Frequency Distribution and then calculate Mode in usual manner.

Age in years (X)	No. of Persons (f)
0 – 10	15
10 – 20	17
20 – 30	19
30 – 40	27
40 – 50	19
50 – 60	13

By inspection, it is clear that modal class is 30–40.

$l_1 = 30$ $f_1 = 27$ $f_0 = 19$ $f_2 = 19$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 30 + \frac{27 - 19}{2 \times 27 - 19 - 19} \times 10 = 35 \text{ years}$$

Cumulative Frequency Distribution
(More Than Series)

Example 50. Calculate Mean, Median and Mode:

Marks (More than)	0	10	20	30	40	50
No. of Students	90	87	78	60	30	12

MEAN: Convert it into Simple Frequency Distribution and then calculate Mean in usual manner.

Marks (X)	No. of Students (f)	Mid-value (m)	$d = m - A$ (A = 25)	$d' = \frac{m-A}{C}$ (C = 10)	fd'
0 – 10	3	5	–20	–2	–6
10 – 20	9	15	–10	–1	–9
20 – 30	18	25	0	0	0
30 – 40	30	35	10	1	30
40 – 50	18	45	20	2	36
50 – 60	12	55	30	3	36
	$\Sigma f = 90$				$\Sigma fd' = 87$

$$\text{Mean } (\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 25 + \frac{87}{90} \times 10 = 34.67 \text{ marks}$$

MEDIAN: Convert it into Simple Frequency Distribution and then calculate Median in usual manner.

Marks (X)	No. of Students (f)	c.f.
0 – 10	3	3
10 – 20	9	12
20 – 30	18	30
30 – 40	30	60
40 – 50	18	78
50 – 60	12	90
	$N = \Sigma f = 90$	

$Me = \frac{N}{2} = \frac{90}{2} = 45^{th}$ item; 45^{th} item lies in group 30 – 40

$l_1 = 30$ c.f. = 30 f = 30 i = 10

$$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 30 + \frac{45 - 30}{30} \times 10 = 35 \text{ marks}$$

MODE: Convert it into Simple Frequency Distribution and then calculate Mode in usual manner.

Marks (X)	No. of Students (f)
0 – 10	3
10 – 20	9
20 – 30	18
30 – 40	30
40 – 50	18
50 – 60	12

By inspection, it is clear that modal class is 30–40.

$l_1 = 30$ $f_1 = 30$ $f_0 = 18$ $f_2 = 18$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 30 + \frac{30 - 18}{2 \times 30 - 18 - 18} \times 10 = 35 \text{ marks}$$

Example 51. Calculate Mean, Median and Mode from the following data:

Mid-value	35	45	55	65	75	85
Frequency	2	18	24	20	8	3

MEAN: Convert it into Simple Frequency Distribution and then calculate Mean in usual manner.

Mid-value (m)	Frequency (f)	d = m − A (A = 55)	$d' = \frac{m-A}{C}$ (C = 10)	fd′
35	2	−20	−2	−4
45	18	−10	−1	−18
55	24	0	0	0
65	20	10	1	20
75	8	20	2	16
85	3	30	3	9
	Σf = 75			Σfd′ = 23

$$\text{Mean } (\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 55 + \frac{23}{75} \times 10 = 58.06$$

MEDIAN: Convert it into Simple Frequency Distribution and then calculate Median in usual manner.

Class-interval (X)	Frequency (f)	c.f.
30 – 40	2	2
40 – 50	18	20
50 – 60	24	44
60 – 70	20	64
70 – 80	8	72
80 – 90	3	75
	N = Σf = 75	

$Me = \frac{N}{2} = \frac{75}{2} = 37.5^{th}$ item; 37.5th item lies in group 50 – 60

$l_1 = 50$ c.f. = 20 f = 24 i = 10

$$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 50 + \frac{37.5 - 20}{24} \times 10 = 57.29$$

MODE: Convert it into Simple Frequency Distribution and then calculate Mode in usual manner.

Class-interval (X)	Frequency (f)
30 – 40	2
40 – 50	18
50 – 60	24
60 – 70	20
70 – 80	8
80 – 90	3

By inspection, it is clear that modal class is 30–40 as frequency of this class is maximum, i.e. 27.

$l_1 = 50$ $f_1 = 24$ $f_0 = 18$ $f_2 = 20$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 50 + \frac{24 - 18}{2 \times 24 - 18 - 20} \times 10 = 56$$

Example ... ...and Mode from the following data:

Marks	10 – 19	20 – 29	30 – 39	40 – 49	50 – 59
No. of Students	3	5	9	3	2

MEAN: Convert it into Simple Frequency Distribution and then calculate Mean in usual manner.

Marks (X)	No. of Students (f)	Mid-value (m)	d = m − A (A = 34.5)	$d' = \frac{m-A}{C}$ (C = 10)	fd′
10 – 19	3	14.5	−20	−2	−6
20 – 29	5	24.5	−10	−1	−5
30 – 39	9	34.5	0	0	0
40 – 49	3	44.5	10	1	3
50 – 59	2	54.5	20	2	4
	Σf = 22				Σfd′ = −4

$$\text{Mean } (\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 34.5 + \frac{-4}{22} \times 10 = 32.68 \text{ marks}$$

MEDIAN: Convert it into Simple Frequency Distribution and then calculate Median in usual manner.

Marks (X)	No. of Students (f)	c.f.
9.5 – 19.5	3	3
29.5 – 29.5	5	8
29.5 – 39.5	9	17
39.5 – 49.5	3	20
49.5 – 59.5	2	22
	N = Σf = 22	

$Me = \frac{N}{2} = \frac{22}{2} = 11^{th}$ item; 11th item lies in group 29.5 – 39.5

$l_1 = 29.5$ c.f. = 8 f = 9 i = 10

$$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 29.5 + \frac{11 - 8}{9} \times 10 = 32.83 \text{ marks}$$

MODE: Convert it into Simple Frequency Distribution and then calculate Mode in usual manner.

Marks (X)	No. of Students (f)
9.5 – 19.5	3
29.5 – 29.5	5
29.5 – 39.5	9
39.5 – 49.5	3
49.5 – 59.5	2

By inspection, it is clear that modal class is 29.5–39.5 as frequency of this class is maximum, i.e. 9.

$l_1 = 29.5$ $f_1 = 9$ $f_0 = 5$ $f_2 = 3$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 29.5 + \frac{9 - 5}{2 \times 9 - 5 - 3} \times 10 = 33.5 \text{ marks}$$

Example 53: ... following data:

Class-interval	Less than 20	20 – 30	30 – 40	40 – 50	50 – 60	Above 60
Frequency	8	12	20	10	6	4

MEAN: Missing class limits are assumed in the following manner:

Class-interval (X)	Frequency (f)	Mid-value (m)	d = m − A (A = 45)	$d' = \frac{m-A}{C}$ (C = 10)	fd′
10 – 20	8	15	−30	−3	−24
20 – 30	12	25	−20	−2	−24
30 – 40	20	35	−10	−1	−20
40 – 50	10	45	0	0	0
50 – 60	6	55	10	1	6
60 – 70	4	65	20	2	8
	Σf = 60				Σfd′ = −54

$$\text{Mean } (\bar{X}) = A + \frac{\Sigma fd'}{\Sigma f} \times C = 45 + \frac{-54}{60} \times 10 = 36$$

MEDIAN: There is no need to determine missing limits, i.e., Median is calculated in the usual manner:

Class-interval (X)	Frequency (f)	c.f.
Less than 20	8	8
20 – 30	12	20
30 – 40	20	40
40 – 50	10	50
50 – 60	6	56
Above 60	4	60
	N = Σf = 60	

$Me = \frac{N}{2} = \frac{60}{2} = 30^{th}$ item; 30th item lies in group 30 – 40

$l_1 = 30$ c.f. = 20 f = 20 i = 10

$$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 30 + \frac{30 - 20}{20} \times 10 = 35$$

MODE: To calculate mode, there is no need to complete the class-intervals.

Class-interval (X)	Frequency (f)
Less than 20	8
20 – 30	12
30 – 40	20
40 – 50	10
50 – 60	6
Above 60	4

By inspection, it is clear that modal class is 30–40 and frequency of this class is 20.

$l_1 = 30$ $f_1 = 20$ $f_0 = 12$ $f_2 = 10$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 30 + \frac{20 - 12}{2 \times 20 - 12 - 10} \times 10 = 34.44$$

...ual Class-intervals

Example 54. Calculate Mean, Median and Mode from the following data:

Class-interval	0–10	10–15	15–20	20–30	30–40	40–50
No. of Students	2	3	2	12	4	7

MEAN: Convert it into Simple Frequency Distribution and then calculate Mean in usual manner.

Marks (X)	No. of Students (f)	Mid-value (m)	fm
0 – 10	2	5	10
10 – 15	3	12.5	37.5
15 – 20	2	17.5	35
20 – 30	12	25	300
30 – 40	4	35	140
40 – 50	7	45	315
	Σf = 30		Σfm = 837.5

$$\text{Mean } (\bar{X}) = \frac{\Sigma fm}{\Sigma f} = \frac{837.5}{30} = 27.92$$

MEDIAN: Convert it into Simple Frequency Distribution and then calculate Median in usual manner.

Marks (X)	No. of Students (f)	c.f.
0 – 10	2	2
10 – 15	3	5
15 – 20	2	7
20 – 30	12	19
30 – 40	4	23
40 – 50	7	30
	N = Σf = 30	

$Me = \frac{N}{2} = \frac{30}{2} = 15^{th}$ item; 15th item lies in group 20 – 30

$l_1 = 20$ c.f. = 7 f = 12 i = 10

$$Me = l_1 + \frac{N/2 - c.f.}{f} \times i = 20 + \frac{15 - 7}{12} \times 10 = 26.67$$

MODE: Convert it into Simple Frequency Distribution and then calculate Mode in usual manner.

Marks (X)	No. of Students (f)
0 – 10	2
10 – 20	5
20 – 30	12
30 – 40	4
40 – 50	7

By inspection, it is clear that modal class is 20–30 as frequency of this class is maximum, i.e. 12.

$l_1 = 20$ $f_1 = 12$ $f_0 = 5$ $f_2 = 3$ $i = 10$

$$Z = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i = 20 + \frac{12 - 5}{2 \times 12 - 5 - 4} \times 10 = 24.67$$

FORMULAE AT A GLANCE

1. MEDIAN **Individual Series**	Me = Size of $\left(\frac{N+1}{2}\right)^{th}$ item *{In Odd Number series}* Average of two items lying on either side of $\left(\frac{N+1}{2}\right)^{th}$ *{In Even Number series}*
Discrete Series	Me = Size of $\left(\frac{N+1}{2}\right)^{th}$ item
Continuous Series	Determine Median Class as $\left[\frac{N}{4}\right]^{th}$ item and apply the formula: $Me = l_1 + \frac{\frac{N}{2} - c.f.}{f} \times i$
2. LOWER QUARTILE **Individual Series**	Q_1 = Size of $\left(\frac{N+1}{4}\right)^{th}$ item
Discrete Series	Q_1 = Size of $\left(\frac{N+1}{4}\right)^{th}$ item
Continuous Series	Determine Quartile Class as $\left[\frac{N}{4}\right]^{th}$ item and apply the formula: $Q_1 = l_1 + \frac{\frac{N}{4} - c.f.}{f} \times i$
3. UPPER QUARTILE **Individual Series**	Q_3 = Size of $3\left(\frac{N+1}{4}\right)^{th}$ item
Discrete Series	Q_3 = Size of $3\left(\frac{N+1}{4}\right)^{th}$ item
Continuous Series	Determine Quartile Class as $3\left[\frac{N}{4}\right]^{th}$ item and apply the formula: $Q_3 = l_1 + \frac{\frac{3N}{4} - c.f.}{f} \times i$

4. MODE **Individual Series**	Mode is the value, which occurs largest number of times.
Discrete Series	If the frequencies are regular and homogeneous and there is a single maximum frequency, then Mode is the value corresponding to the highest frequency (Otherwise use Grouping Method)
Continuous Series	*Step 1:* Determine the Modal Class: (i) By inspection, if frequencies are regular, homogeneous and there is a single maximum frequency; Otherwise (ii) Grouping Method. *Step 2:* Apply the following formula: $Mo = l_1 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times i$

Abbreviations of Mode, Median, Lower Quartile and Upper Quartile

Me = Median

Q_1 = Lower Quartile

Q_3 = Upper Quartile

l_1 = Lower limit of the median class or Quartile class or modal class

c.f. = Cumulative frequency of class preceding median or Quartile class

f = Simple frequency of the median or Quartile class

i = Class-interval of the median class or Quartile class or modal class

N = Number of items

Mo = Mode

f_1 = Frequency of the modal class

f_0 = Frequency of the class preceding the modal class

f_2 = Frequency of the class succeeding the modal class