Inventory Management Example Problems with Solutions

1. An auto parts supplier sells Hardy-brand batteries to car dealers and auto mechanics. The annual demand is approximately 1,200 batteries. The supplier pays \$28 for each battery and estimates that the annual holding cost is 30 percent of the battery's value. It costs approximately \$20 to place an order (managerial and clerical costs). The supplier currently orders 100 batteries per month.

   a. Determine the ordering, holding, and total inventory costs for the current order quantity.

   b. Determine the economic order quantity (EOQ).

   c. How many orders will be placed per year using the EOQ?

   d. Determine the ordering, holding, and total inventory costs for the EOQ. How has ordering cost changed? Holding cost? Total inventory cost?

   $\boxed{\textbf{Solution}}$ We are given the following information:

   annual demand: $D = 1200$ batteries per year

   item cost: $c = \$28$ per battery

   holding cost: $H = ic = 0.30(28) = \$8.40$ per battery per year

   order cost: $S = \$20$ per order

   current order quantity: $Q = 100$ batteries

   a. The current ordering and holding costs are: $\frac{D}{Q}S + \frac{Q}{2}H = \frac{1200}{100}(20) + \frac{100}{2}(8.40) = 240 + 420 = \$660$.

   b. The EOQ is $Q^* = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2 \times 1200 \times 20}{8.40}} = 75.6 \to 76$ batteries.

   c. The company will place $\frac{D}{Q^*} = \frac{1200}{76} = 15.8$ orders per year.

   d. The new ordering and holding costs are: $\frac{D}{Q^*}S + \frac{Q^*}{2}H = \frac{1200}{76}(20) + \frac{76}{2}(8.40) = 315.79 + 319.20 = \$634.99$. The company will save \$25.01 by using the EOQ.

2. Upon closer inspection, the supplier determines that the demand for batteries is normally distributed with mean 4 batteries per day and standard deviation 3 batteries per day. (The supplier is open 300 days per year.) It usually takes about 4 days to receive an order from the factory.

   a. What is the standard deviation of usage during the lead time?

   b. Determine the reorder point needed to achieve a service level of 95 percent.

   c. What is the safety stock? What is the holding cost associated with this safety stock?

   d. How would your analysis change if the service level changed to 98 percent?

   $\boxed{\textbf{Solution}}$ In addition to the information from the problem above, we are told:

   average demand rate: $d = 4$ batteries per day

   standard deviation of demand: $\sigma_d = 3$ batteries *per day*

   lead time: $L = 4$ days

   300 operating days per year

a. The standard deviation of demand during the lead time is $\sigma_L = \sigma_d \sqrt{L} = 3(\sqrt{4}) = 6$ batteries.

b. The reorder point is equal to the average demand expected during the lead time plus some safety stock. The service level is 95 percent. Examining the Table of Normal Demand Percentages (see the last page of this handout), we see that this corresponds to $z = 1.65$ (take the average of $z = 1.6$ and $z = 1.7$, since 95 percent is between 94.5 and 95.5 percent). In other words, we need to keep 1.65 standard deviations worth of extra inventory on hand to ensure that the probability of running out is less than 5 percent. Now we can figure out the reorder point: $R = dL + z\sigma_L = 4(4) + 1.65(6) = 16 + 9.9 = 25.9 \rightarrow 26$ batteries. We place an order for 76 batteries when the inventory level drops to 26 batteries.

c. The safety stock is the inventory in excess of the expected demand during the lead time. In other words, the safety stock is $26 - 16 = 10$ batteries. The associated holding cost is simply $10 \times H = 10 \times 8.40 = \$84.00$.

d. If the service level changes to 98 percent, then we must go back and determine a new z. Consulting the table, we see that 98 percent is between 97.7 and 98.2 percent. Averaging the two corresponding z values gives us $z = 2.05$ for a service level of 98 percent. Thus, the reorder point will be: $R = dL + z\sigma_L = 4(4) + 2.05(6) = 16 + 12.3 = 28.3 \rightarrow 29$ batteries. Place an order for 76 units when the inventory level drops to 29 units.

3. Foster Drugs, Inc., handles a variety of health and beauty aid products. A particular hair conditioner product costs Foster \$2.95 per unit. The annual holding cost rate is 20 percent. Using an EOQ model, they determined that an order quantity of 300 units should be used. The lead time to receive an order is one week, and the demand is normally distributed with a mean of 150 units per week and a standard deviation of 40 units per week.

a. What is the reorder point if the firm is willing to tolerate a 1-percent chance of a stockout during an order cycle?

b. What safety stock and annual safety stock cost are associated with your recommendation in part a?

c. Foster is considering making a transition to a periodic-review system in an attempt to coordinate ordering of some of its products. The review period would be two weeks and the delivery lead time would remain one week. What target inventory level would be needed to ensure the same 1-percent risk of stockout?

d. What is the safety stock associated with your answer to part c? What is the annual cost associated with holding this safety stock?

e. Compare your answers to parts b and d. If you were the manager of Foster Drugs, would you choose a continuous- or periodic-review system?

$\boxed{\text{Solution}}$ We are given the following information:

item cost: $c = \$2.95$ per unit

carrying "interest rate": $i = 0.20$ per unit per year

lead time: $L = 1$ week

average demand rate: $d = 150$ units per week

standard deviation of demand: $\sigma_d = 40$ units per week

current order quantity: $Q = 300$ units

a. A 1-percent stockout risk corresponds to a service level of 99 percent. Consulting the table, we see that this service level corresponds to $z = 2.3$. First, compute the mean and standard deviation of demand during the lead time: $dL = 150(1) = 150$ and $\sigma_L = \sigma_d\sqrt{L} = 40\sqrt{1} = 40$. Now we have all of the information necessary to determine the reorder point: $R = dL + z\sigma_L = 150 + 2.3(40) = 150 + 93.05 = 243.05 \rightarrow 243$ units.

b. The safety stock is 93 units, and the holding cost associated with the safety stock is $93 \times H = 93 \times ic = 93 \times 0.20(2.95) = \$54.87$.

c. We are told that the review period is two weeks so $P = 2$. Use this number, along with $L = 1$ to compute the mean and standard deviation of demand during the lead time and the review period: $d(P + L) = 150(2 + 1) = 450$ and $\sigma_{P+L} = \sigma_d\sqrt{P + L} = 40\sqrt{2 + 1} = 69.28$. The z value remains the same. The target inventory level is: $T = d(P + L) + z\sigma_{P+L} = 450 + 2.3(69.28) = 450 + 159.35 = 609.35 \rightarrow 610$ units.

d. The safety stock is 160 units (rounded up), and the holding cost associated with the safety stock is $160 \times H = 160 \times ic = 160 \times 0.20(2.95) = \$94.40$.

e. The periodic review method requires a larger safety stock, so it costs more: $94.40 - 54.87 = \$39.53$. However, using a fixed-period method facilitates ordering multiple items from a single supplier, so it is probably worth the extra cost.

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Formulas for Inventory Management Problems

Fixed-Quantity System (Q System)

$$\text{EOQ: } Q^* = \sqrt{\frac{2DS}{H}} \qquad \text{TC} = \frac{D}{Q}S + \frac{Q}{2}H \qquad H = ic$$

$$R = dL + z\sigma_L \qquad \sigma_L = \sigma_d\sqrt{L}$$

Fixed-Period System (P System)

$$T = d(P + L) + z\sigma_{P+L} \qquad \sigma_{P+L} = \sigma_d\sqrt{P + L}$$

Table of Normal Demand Percentages

z	Service Level (%)	Stockout Risk (%)
0.0	50.0	50.0
0.5	69.1	30.9
1.0	84.1	15.9
1.1	86.4	13.6
1.2	88.5	11.5
1.3	90.3	9.7
1.4	91.9	8.1
1.5	93.3	6.7
1.6	94.5	5.5
1.7	95.5	4.5
1.8	96.4	3.6
1.9	97.1	2.9
2.0	97.7	2.3
2.1	98.2	1.8
2.2	98.6	1.4
2.3	98.9	1.1
2.4	99.2	0.8
2.5	99.4	0.6
2.6	99.5	0.5
2.7	99.6	0.4
2.8	99.7	0.3
2.9	99.8	0.2
3.0	99.9	0.1