Chapter 16

Forecasting and Time-series Analysis

I have but one lamp by which my feet are guided, and that is the lamp of experience. I know of no way of judging the future but the past.

—Patrick Henry

The penguin flies backwards because he does not care to see where he's going, but wants to see where he's been.

—Fred Allen

LEARNING OBJECTIVES

After studying this chapter, you should be able to

- understand the pattern of the historical data and then extrapolate the pattern into the future.
- understand the different approaches to forecasting that can be applied in business.
- gain a general understanding of time-series forecasting techniques.
- learn how to decompose time-series data into their various components and to forecast by using decomposition techniques.

16.1 INTRODUCTION

The increasing complexity of the business environment together with changing demands and expectations are compelling every organization to understand consequences of its decisions (actions or strategies) on the future businesses or services. The knowledge of forecasting methods is essential for decision makers to make reliable and accurate estimates and assess or evaluate the future consequences of decisions in the face of uncertainty.

A flow chart of forecasts and the decision-making process is shown in Fig. 16.1. In general, the decisions are influenced by the chosen strategy with regard to an organization's future priorities and activities. Once decisions are taken, the consequences are measured in terms of expectation to achieve the desired products/services levels.

Decisions also get influenced by the additional information obtained from the forecasting method used. Such information and the accuracy of the forecasts may also affect the strategy formulation of an organization. Thus, an organization needs to establish a monitoring system to compare planned performance with the actual. Divergence, if any, and no matter what is the cause of such divergence, should be fed back into the forecasting process, to generate new forecasts. A few objectives of forecasting are as follows:

- The creation of plans of action because it is not possible to evolve a system of business control without an acceptable system of forecasting.
- Monitoring of the progress of action plans continuously based on forecasts.
- Developing a warning system of the critical factors because they might drastically affect the performance of the plan.

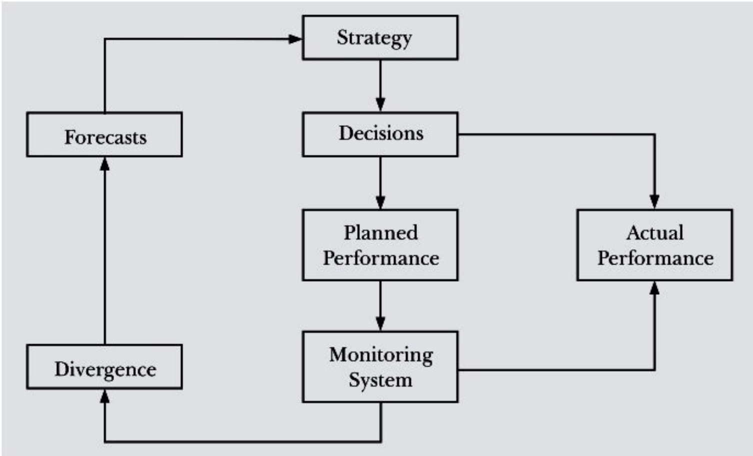


Figure 16.1 Decision-making Process and Forecasts

16.2 TYPES OF FORECASTS

The objectives of any organization are facilitated by a number of different types of forecasts. These may be related to cash flows, operating budgets, personnel requirement, inventory levels, and so on. However, a broad classification of the types of forecasts is as follows:

Demand Forecasts: These are concerned with the predictions of demand for products and/or services based on upon sales and marketing information. These forecasts facilitate in formulating material and capacity plans, and also serve as inputs to financial, marketing and manpower planning.

Environmental Forecasts: These are concerned with the social, political and economic environments of the state and/or the country. Economic forecasts are helpful in predicting inflation rates, money supplies, operating budget and so on.

Technological Forecasts: These are concerned with new developments in existing technologies. The technological forecast is important for technologically advanced companies dealing with computers, aerospace, nuclear and so on.

16.3 TIMING OF FORECASTS

Forecasts are usually classified according to time period and use. The three broad categories of forecasts are as follows:

Short-range Forecast: The short-range forecast has a time-span of up to one year but usually is less than three months. It is normally used for job scheduling, work force levels, job assignments and production levels.

Medium-range Forecast: The medium-range forecast has a time-span from one to three years. It is normally used for sales planning, production planning, budgeting and so on.

Long-range Forecast: The long-range forecast has a time-span of three or more years. It is used for designing and installing new plants, facility location, capital expenditures, research and development and so on.

The medium- and long-range forecasts differ from short-range forecast on account of following three factors:

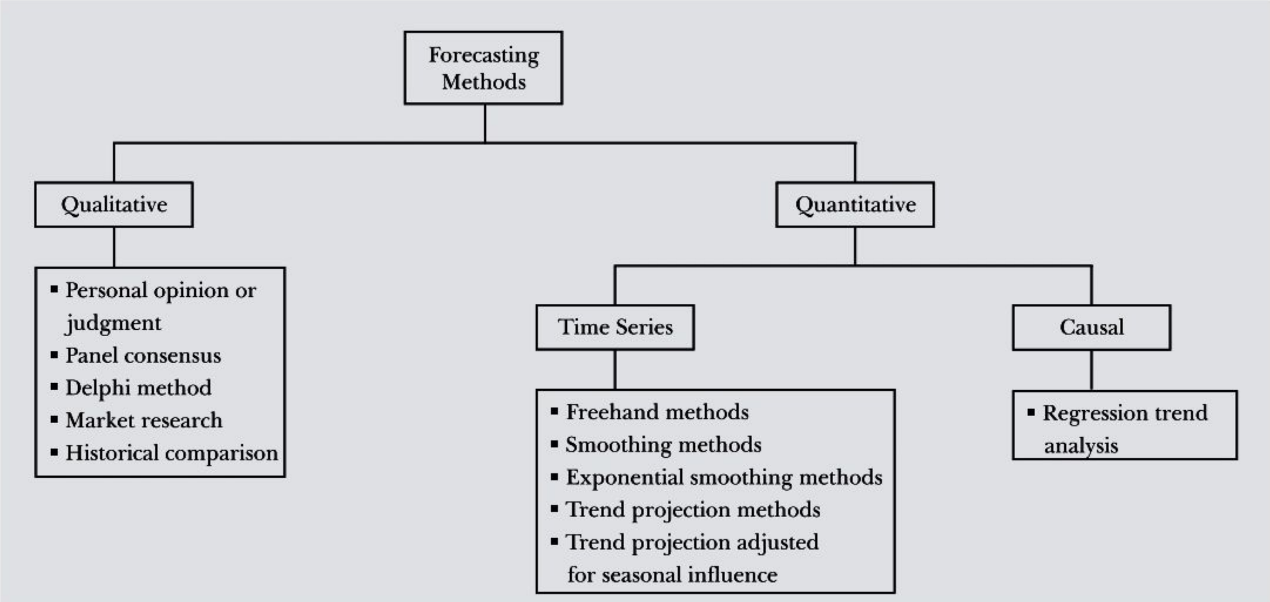
(i) Medium- and long-range forecasts deal with more comprehensive issues and support decisions regarding design and development of new products, plants and processes.
(ii) Mathematical techniques such as moving averages, exponential smoothing and trend extrapolation are used for short-range forecasts.

(iii) The short-range forecasts tend to be more accurate than long-range forecasts. For example, sales forecasts need to be updated regularly in order to maintain proper inventory level of products. After each sales period, the forecast should be reviewed and revised.

16.4 FORECASTING METHODS

Forecasting methods may be classified as either quantitative or qualitative (opinion or judgmental) as shown in Figure 16.2.

Figure 16.2
Forecasting Methods



16.4.1 Quantitative Forecasting Methods

These methods are applied when

(i) past data about the variable being forecast is available,
(ii) information can be quantified, and
(iii) pattern of the past will continue into the future.

The quantitative methods of forecasting are further classified into two categories:

1. **Time-series Forecasting Methods:** A time-series is a set of measurements of a variable that changes through time. The time variable fluctuates uniformly in the same direction from past to future rather than arbitrarily. Thus, there is a freedom to choose the time periods at which observations can be made. The time-series data are gathered on a variable characteristic over a period of time at regular intervals.

   The time-series forecasting methods attempt to predict the outcome for a future time period by analysing patterns, cycles or trends over a period of time.
2. **Causal Forecasting Methods:** The causal forecasting methods are based on the assumptions that the variable value to be forecasted has a cause-effect relationship with one or more other variables. A linear regression analysis is one of the causal forecasting methods.

Causal Forecasting Methods: Forecasting methods that relate a time-series to other variables which are used to explain cause and effect relationship.

16.4.2 Qualitative Forecasting Methods

The qualitative forecasting methods are used for collecting opinions and judgments of individuals who are expected to have the best knowledge of current activities or future plans of the organization. For example, marketing professionals through regular contact with customers are presumably familiar with retail market segment, trends by product line, demand trend and so on.

In qualitative forecasting methods, decision makers can incorporate subjective experience as inputs along with objectives data. Since each human being has different knowledge, experience and perspective of reality, intuitive forecasts are likely to differ from one individual to another individual. The quantification of data gives decision makers a more precise meaning than words which are inexact and capable of being misunderstood. The following are few qualitative forecasting methods:

(i) **Personal Opinion:** In this approach, an individual does some forecast about a variable of interest based on his/her own judgment or opinion without using a formal quantitative model. Such forecast can be relatively reliable and accurate.

This approach is usually recommended when conditions in the present are not likely to hold in the future. For example, an assessment whether inventory levels are likely to last until the next replenishment; a machine will require repair in the next month and so on.

(ii) **Panel Consensus:** In this approach, it is possible to develop consensus among group of individuals to reduce the prejudices and ignorance that may arise in the individual judgment. Such a panel of individuals is encouraged to share information, opinions and assumptions (if any) to predict the future value of a variable of interest.

The disadvantage of this method is that it is dependent on group dynamics and frequently requires a facilitator or convener to coordinate the process of developing a consensus.

(iii) **Delphi Method:** In this method, a panel of experts uses the collective experience and judgment. The panel members may be located in different places, never meet and do not know each other. Each member is given a questionnaire to complete relating to the area under investigation. Based on the responses in questionnaire form from members, a summary is prepared and a copy of it is sent to each member for revision of responses, if any, based on the summary report. This process of updating the summary report is repeated until the desirable consensus is reached among members. This method produces a narrow range of forecasts rather than a single view of the future.

Delphi Method: A quantitative forecasting method that obtains forecasts through group consensus.

(iv) **Market Research:** This method is used to collect data based on well-defined objectives and assumptions about the future value of a variable. For market research, a questionnaire related to the subject of interest is distributed among respondents. A summary report based on the responses in questionnaire form from respondents is prepared to develop survey results.

(v) **Historical Comparison:** In this method, the data are arranged chronologically and the time-series approach is used to facilitate comparison between one time period and to the next. It provides a basis for making comparisons by isolating the effects of various influencing factors on the patterns of variable values.

16.5 STEPS OF FORECASTING

The following are the general steps to present a systematic procedure of initiating, designing and implementing a forecasting system:

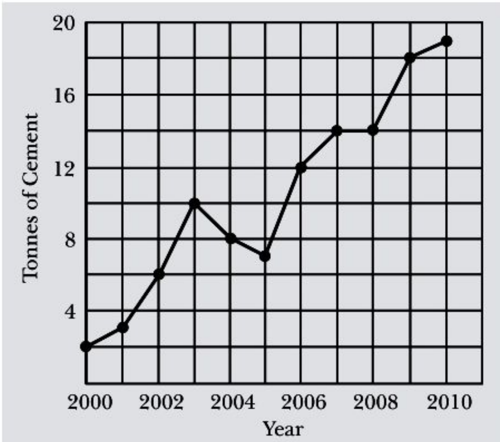
1. Define organization's objectives of forecasting in order to make use of the best available information to guide future activities and policies to be achieved.
2. Select the variables to be forecasted such as capital investment, employment level, inventory level and purchasing of new equipment.
3. Determine the time horizon—short, medium or long term—of the forecast in order to predict changes which may follow the present level of activities.
4. Select an appropriate forecasting method to make projections of the future keeping in view the reasons of changes in the past.
5. Collect the relevant data required for forecasting.
6. Make the forecast and implement its results.

16.6 TIME-SERIES ANALYSIS

A time-series is a set of numerical values of some variable obtained at regular period over time. These numerical values are usually tabulated or graphed to understand the behaviour of the variable. Figure 16.3 presents the export of cement (in tons) by a cement company between 1994 and 2004. The graph suggests that the series is time dependent. Through such a graph, the management of the company may determine time dependence of the series and develop a procedure to predict the future levels with some degree of reliability. The nature of the time dependence is often analysed by decomposing the time-series into its components.

Figure 16.3
Export of Cement

Year	*Export (tonnes)*
2000	2
2001	3
2002	6
2003	10
2004	8
2005	7
2006	12
2007	14
2008	14
2009	18
2010	19



Time-series: A set of observations measured at successive points in time or over successive periods of time.

16.6.1 Objectives of Time-series Analysis

1. In time-series analysis, it is assumed that the various factors which have already influenced the patterns of change in the value of the variable under study will continue to do so almost in the same manner in future also. Thus, one of the objectives of time-series analysis is to identify the pattern and isolate the influencing factors (or effects) for prediction, planning and control of future values of the variable.
2. The review and evaluation of progress in any phenomenon are made based on time-series data. For example, evaluation of the policy of controlling inflation and price rise is done based on various price indices that are based on the analysis of time-series.

16.6.2 Time-series Patterns

In time-series, it is assumed that the data consist of a pattern along with random fluctuations. This may be expressed in the following form:

Trend: A type of variation in time-series that reflects a long-term movement in time-series over a long period of time.

Actual value of the variable at time t = Mean value of the variable at time t + Random deviation from mean value of the variable at time t

$$\hat{y} = \text{Pattern} + e$$

where $\hat{y}$ is the forecast variable at period t, pattern is the mean value of the forecast variable at period t, and e is the random fluctuation from the pattern that occurs of the forecast variable at period t.

16.6.3 Components of a Time-series

The **time-series** data contain four components: *trend*, *cyclicality*, *seasonality* and *irregularity*. Not all time-series have all these components. Figure 16.4 shows the effects of these time-series components over a period of time.

Trend

Sometimes a time-series displays a steady tendency of either upward or downward movement in the average (or mean) value of the forecast variable y over time. Such a tendency is called a trend. When observations are plotted against time, a straight line describes the increase or decrease in the time-series over a period of time.

Cycles

Upward and downward movements in the variable value about the trend time over a time period are called cycles. A business cycle may vary in length, usually more than a year but less than 5 to 7 years. The movement is through four phases from *peak* (prosperity) to *contradiction* (recession) to *trough* (depression) to *expansion* (recovery or growth) as shown in Fig. 16.4.

Cyclical Variation: A type of variation in time-series, in which the value of the variable fluctuates above and below a trend line and lasting more than one year.

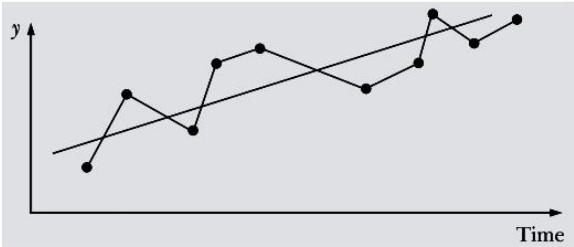


Figure 16.4
Time-series Effects

Seasonal

It is a special case of a cycle component of time-series in which fluctuations are repeated usually within a year (e.g. daily, weekly, monthly and quarterly) with a high degree of regularity. For example, average sales for a retail store may increase greatly during festival seasons.

Seasonal Variation: A type of variation in time-series that shows a periodic pattern of change in time-series within a year; patterns tend to be repeated from year to year.

Irregular

Variations are rapid charges or *bleeps* in the data caused by short-term unanticipated and non-recurring factors. Irregular fluctuations can happen as often as day to day.

Irregular Variation: A type of variation in time-series that reflects the random variation of the time-series values which is completely unpredictable.

16.7 TIME-SERIES DECOMPOSITION MODELS

The analysis of time-series consists of two major steps:

(i) Identifying the various factors or influences which produce the variations in the time-series.
(ii) Isolating, analysing and measuring the effect of these factors independently holding other things constant.

The purpose of decomposition is to break a time-series into its components: trend (T), cyclical (C), seasonality (S) and irregularity (I). Such decomposition helps to isolate influence of each of the four components on the actual series and becomes the basis for forecasting. Two commonly used models for decomposition of a time-series are discussed below.

16.7.1 Multiplicative Model

The actual values of a time-series, Y can be found by multiplying its four components at a particular time period. The effect of four components on the time-series is interdependent. The multiplicative time-series model is defined as

$$Y = T \times C \times S \times I \leftarrow \text{Multiplicative model}$$

This model is useful in situations where the effect of C, S and I is measured in relative sense instead of absolute sense. The geometric mean of C, S and I is assumed to be less than one. For example, suppose sales of a product for a period of 20 months is Y_{20} = 423.36. Decomposing sales into its components as trend component (mean sales) 400; effect of current cycle (0.90) which decreases sales by 10 per cent; and seasonality of the series (1.20) that increases sales by 20 per cent. In the absence of random fluctuation, the expected value of sales for the given period is 400 × 0.90 × 1.20 = 432. If the random factor decreases sales by 2 per cent in this period, then the actual sales volume will be 432 × 0.98 = 423.36.

16.7.2 Additive Model

In this model, it is assumed that the effect of various components on a time-series can be estimated by adding these components. The additive time-series model is defined as

$$Y = T + C + S + I \leftarrow \text{Additive model}$$

where C, S and I are absolute quantities and can have positive or negative values. It is assumed that these four components are independent of each other.

Conceptual Questions 16A

1. Briefly describe the steps that are used to develop a forecasting system.
2. What is forecasting? Discuss in brief the various theories and methods of business forecasting. *[Delhi Univ., MBA, 2005]*
3. For what purpose do we apply time-series analysis to data collected over a period of time?
4. How can one benefit from determining past patterns?
5. What is the difference between a causal model and a time-series model?
6. What is a judgmental forecasting model, and when is it appropriate?
7. Explain clearly the different components into which a time-series may be analysed. Explain any method for isolating trend values in a time-series.
8. Explain what you understand by time-series. Why is time-series considered to be an effective tool of forecasting?
9. Explain briefly the additive and multiplicative models of time-series. Which of these models is more popular in practice and why? *[Osmania Univ., MBA, 2008]*
10. Identify the four principal components of a time-series and explain the kind of change, over time, to which each applies.
11. What is the advantage of reducing a time-series into its four components?
12. Despite great limitations of statistical forecasting, forecasting techniques are invaluable to the economist, the businessman, and the government. Explain.
13. (a) Why are forecasts important to organizations?
    (b) Explain the difference between the terms: seasonal variation and cyclical variation.
    (c) Give reasons why the seasonal component in the time-series is not constant? Give examples where you believe the seasonality may change.
14. Identify the classical components of a time-series and indicate how each is accounted for in forecasting.

16.8 QUANTITATIVE FORECASTING METHODS

The quantitative forecasting methods are classified into two general categories:

1. **Time-series Methods:** This method takes into consideration an observed historical pattern for any variable and projects the same into the future using a mathematical formula. These methods do not suggest that a variable will take some future value.
2. **Causal Methods:** In this method, regression analysis and correlation analysis identify factors that influence or cause variation in the value of any variable in some predictable manner.

16.8.1 Freehand Method

A freehand curve drawn through the data values is an easy and adequate representation of time-series. In Fig. 16.3, a straight line connecting the 1997 and 2004 exports volumes is fairly a good representation of the given data.

The forecast is done by extending the trend line. A trend line fitted by the freehand method should confirm to the following conditions:

(i) The trend line should be smooth.
(ii) The sum of the vertical deviations of the observations above the trend line should equal the sum of the vertical deviations of the observations below the trend line.
(iii) The sum of squares of the vertical deviations of the observations from the trend line should be as small as possible.
(iv) The trend line should bisect the cycles so that area above and below the trend line for each full cycle.

Example 16.1: Fit a trend line to the following data by using the freehand method.

Year	: 1997	1998	1999	2000	2001	2002	2003	2004
Sales turnover : (₹ in lakh)	80	90	92	83	94	99	92	104.

Solution: Freehand graph of sales turnover (₹ in lakh) from 1997 to 2004 is shown in Fig 16.5. Forecast can be obtained simply by extending the trend line.

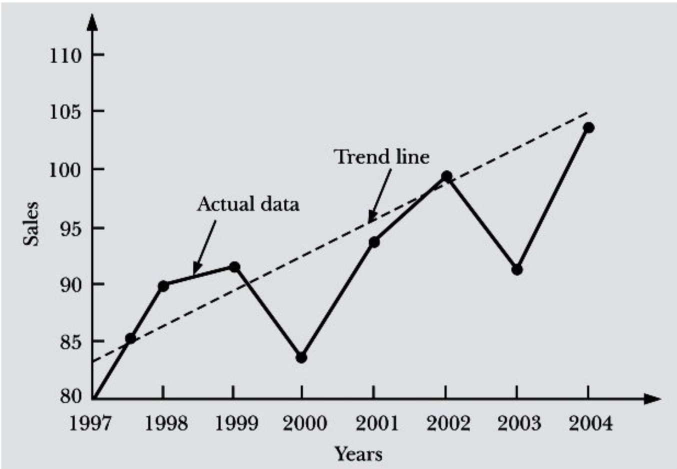


Figure 16.5
Graph of Sales Turnover

Limitations of Freehand Method

(i) This method is highly subjective because the trend line depends on personal judgment and therefore good-fit for one individual may not be same for another.
(ii) The trend line used as a basis for predictions cannot have much value.
(iii) The construction of a freehand trend is very time-consuming provided a careful job is to be done.

16.8.2 Smoothing Methods

The smoothing methods provide pattern of movement in the data over time by eliminating random variations due to irregular components of the time-series. The following three smoothing methods are discussed in this section:

1. Moving averages
2. Weighted moving averages
3. Semi-averages

Moving Averages

Moving Averages: A quantitative method of forecasting or smoothing a time-series by averaging each successive group of data values.

To observe the movement of some variable values over a period of time and then to project this movement into the future, it is essential to smooth out first the irregular pattern in the historical values of the variable, and later use this as the basis for a future projection. This can be done by using the method of **moving averages**.

This is a subjective method and depends on the length of the period for calculating moving averages. To remove the effect of irregular pattern, the period chosen for calculating moving averages should be an integer value. Preferably the period chosen should be a multiple of the estimated average length of a cycle in the series.

The moving averages as an estimator of the value of a variable in the next period given a period of length n is expressed as

$$\text{Moving average, } MA_{t+1} = \frac{\sum\{D_t + D_{t-1} + D_{t-2} + \ldots + D_{t-n+1}\}}{n}$$

where t is the current time period; D is the actual data which is exchanged each period and n is the length of time period.

In this method, the term 'moving' is used because each time an average is computed by summing the values from a given number of periods through deleting the oldest value and adding a new value.

Advantage

The major *advantage* of moving average method is the opportunity to focus on the long-term trend (and cyclical) movements in a time-series without the obscuring effect of short-term influences.

Limitations

(i) As the length of period chosen for computing the averages increases, it smoothens the variations better but it also makes the method less sensitive to real changes in the data.

(ii) It is difficult to choose the optimal length of time for computing the moving average but it cannot be computed for the first $(n - 1)/2$ years or the last $(n - 1)/2$ year of the series.

(iii) Moving averages cannot predict trends very well because averages always stay within past levels and do not predict a change to either a higher or a lower level.

(iv) Moving averages do not usually adjust for such time-series effects as trend, cycle or seasonality.

Example 16.2: Use following data to compute a three-year moving average for all available years. Also determine the trend and short-term error.

Year	*Production (in '000 tonnes)*	*Year*	*Production (in '000 tonnes)*
2000	21	2005	22
2001	22	2006	25
2002	23	2007	26
2003	25	2008	27
2004	24	2009	26

Solution: Computing moving average for the first 3 years as follows:

$$\text{Moving average} = \frac{21 + 22 + 23}{3} = 22$$

This average value of first three-year can be used to forecast the production volume in fourth year, 2008. Since 25,000 tonnes production was made in 2008; therefore, error of the forecast becomes 25,000 – 22,000 = 3000 tonnes.

Similarly, moving average for the year 2006 to 2008 is

$$\text{Moving average} = \frac{22 + 23 + 25}{3} = 23.33$$

Calculations of three-year moving average are shown in Table 16.1.

Table 16.1 Calculations of Trend and Short-term Fluctuations

Year	*Production* y	*3-Year Moving Total*	*3-Yearly Moving Average (Trend values)* $\hat{y}$	*Forecast Error* $(y - \hat{y})$
2000	21	—	—	—
2001	22	(21 + 22 + 23) = 66	66/3 = 22.00	0
2002	23	(22 + 23 + 25) = 70	70/3 = 23.33	–0.33
2003	25	(23 + 25 + 24) = 72	72/3 = 24.00	1.00
2004	24	71	23.67	0.33
2005	22	71	23.67	–1.67
2006	25	73	24.33	0.67
2007	26	78	26.00	0
2008	27	(26 + 27 + 26) = 79	79/3 = 26.33	0.67
2009	26	—	—	—

Odd and Even Numbers of Years If period of length n is an odd number, then moving average period is centred on middle period in the consecutive sequence of n periods. For example, if $n = 5$, then the first five year moving average, $MA_3(5)$ is centred on the third year, $MA_4(5)$ is centred on the fourth year... and $MA_9(5)$ is centred on the ninth year.

Since moving average cannot be obtained for the first $(n - 1)/2$ years or the last $(n - 1)/2$ year of the series; therefore for a 5-year moving average, it cannot be computed for the just two years or the last two years of the series.

If period of length n is an even number, then moving average can be computed by taking an average of each part. For example, if $n = 4$, then the first four year moving average, $M_3(4)$ is an average of the first four year data, and the second moving average $M_4(4)$ is the average of data values 2 through 5.

Example 16.3: Assume a four-year cycle and calculate the trend by the method of moving average from the following data relating to the production of tea in India:

Year	*Production (million lbs)*	*Year*	*Production (million lbs)*
1997	464	2002	540
1998	515	2003	557
1999	518	2004	571
2000	467	2005	586
2001	502	2006	612

[*Madras Univ., M.Com., 2007*]

Solution: The first 4-year moving average is

$$MA_3(4) = \frac{464 + 515 + 518 + 467}{4} = \frac{1964}{4} = 491.00$$

This moving average is centred on the middle value (third year of the series). Similarly,

$$MA_4(4) = \frac{515 + 518 + 467 + 502}{4} = \frac{2002}{4} = 500.50$$

This moving average is centred on the fourth year of the series. The computations of 4-year moving averages are shown in Table 16.2.

Table 16.2 Calculations of Trend and Short-term Fluctuations

Year	*Production (mn lbs)*	*4-Yearly Moving Totals*	*4-Yearly Moving Average*	*4-Yearly Moving Average Centred*
1997	464	—	—	—
1998	515	—	—	—
		1964	491.00	
1999	518			495.75
		2002	500.50	
2000	467			503.62
		2027	506.75	
2001	502			511.62
		2066	516.50	
2002	540			529.50
		2170	542.50	
2003	557			553.00
		2254	563.50	
2004	571			572.50
		2326	581.50	
2005	586	—	—	—
2006	612	—	—	—

Weighted Moving Averages

Weighted Moving Average: A quantitative method of forecasting or smoothing a time-series by computing a weighted average of past data values; sum of weights must equal one.

A moving average where some time periods are weighted differently than others is called a weighted moving average. Choice of weights is arbitrary because there is no set rule to do so. Generally, the most recent observation is assigned larger weightage and the weightage decreases for older data values.

A weighted moving average is computed as

$$\text{Weighted moving average} = \frac{\sum(\text{Weight for period } n)\,(\text{Data value in period } n)}{\sum \text{Weights}}$$

Example 16.4: Vacuum cleaner sales for 12 months are given below. The owner of the supermarket decides to forecast sales by weighting the past three months as follows:

Weight Applied	*Month*
3	Last month
2	Two months ago
1	Three months ago
6	

Months :	1	2	3	4	5	6	7	8	9	10	11	12
Actual sales : (in units)	10	12	13	16	19	23	26	30	28	18	16	14

Solution: The results of 3-month weighted average are shown in Table 16.3

$$\bar{x}_{\text{weighted}} = 3M_{t-1} + 2M_{t-2} + 1M_{t-3}$$

$$= \frac{1}{6}[3 \times \text{Sales last month} + 2 \times \text{Sales two months ago} + 1 \times \text{Sales three months ago}]$$

Table 16.3 Weighted Moving Average

Month	*Actual Sales*	*Three-month Weighted Moving Average*
1	10	—
2	12	—
3	13	—
4	16	$[(3 \times 13) + (2 \times 12) + (1 \times 10)] = \frac{121}{6}$
5	19	$[(3 \times 16) + (2 \times 13) + (1 \times 12)] = \frac{141}{3}$
6	23	$[(3 \times 19) + (2 \times 16) + (1 \times 13)] = 17$
7	26	$[(3 \times 23) + (2 \times 19) + (1 \times 16)] = \frac{201}{2}$
8	30	$[(3 \times 26) + (2 \times 23) + (1 \times 19)] = \frac{235}{6}$
9	28	$[(3 \times 30) + (2 \times 26) + (1 \times 23)] = \frac{271}{2}$
10	18	$[(3 \times 28) + (2 \times 30) + (1 \times 26)] = \frac{289}{3}$
11	16	$[(3 \times 18) + (2 \times 28) + (1 \times 30)] = \frac{231}{3}$
12	14	$[(3 \times 16) + (2 \times 18) + (1 \times 28)] = \frac{182}{3}$

Example 16.5: A food processor uses a moving average to forecast next month's demand. Past actual demand (in units) is shown below:

Month :	43	44	45	46	47	48	49	50	51
Actual demand :	105	106	110	110	114	121	130	128	137

(a) Compute a simple five-month moving average to forecast demand for month 52.
(b) Compute a weighted three-month moving average where the weights are highest for the latest months and descend in order of 3, 2, 1.

Solution: Calculations for five-month moving average are shown in Table 16.4.

Table 16.4 Five-month Moving Average

Month	*Actual Demand*	*5-month Moving Total*	*5-month Moving Average*
43	105	—	—
44	106	—	—
45	110	545	109.50
46	110	561	112.2
47	114	585	117.0
48	121	603	120.6
49	130	630	126.0
50	128	—	—
51	137	—	—

(a) Five-month average demand for month 52 is

$$\frac{\sum x}{\text{Number of periods}} = \frac{114 + 121 + 130 + 128 + 137}{5} = 126 \text{ units}$$

(b) Weighted three-month average as per weights is as follows:

$$\bar{x}_{\text{weighted}} = \frac{\sum \text{Weight} \times \text{Data value}}{\sum \text{weight}}$$

where

Month	Weight × Value = Total
51	3 × 137 = 411
50	2 × 128 = 256
49	1 × 130 = 130
	6 797

$$\bar{x}_{\text{weighted}} = \frac{797}{6} = 133 \text{ units.}$$

Semi-average Method

The semi-average method is used to estimate the slope and intercept of the trend line provided time-series is represented by a linear function. In this method, the data are divided into two parts and their respective arithmetic means are computed. The two arithmetic mean points are plotted corresponding to the midpoint of the class interval covered by the respective part and then these points are joined by a straight line to get the required trend line. The arithmetic mean of the first part is the intercept value, and the slope (change per unit time) is determined by the ratio of the difference in the arithmetic mean of the number of years between them to get a time-series of the form $\hat{y} = a + bx$. The $\hat{y}$ equation should always be stated with reference to the year where $x = 0$ and a description of the units of x and y.

Example 16.6: Fit a trend line to the following data using semi-average method and forecast the sales for the year 2012.

Year	*Sales of Firm (thousand units)*	*Year*	*Sales of Firm (thousand units)*
2003	102	2007	108
2004	105	2008	116
2005	114	2009	112
2006	110		

Solution: Since number of years is odd in number, therefore divide the data into equal parts (A and B) of 3 years ignoring the middle year (2006). The average of part A and B is

$$\bar{y}_A = \frac{102 + 105 + 114}{3} = \frac{321}{3} = 107 \text{ units}$$

$$\bar{y}_B = \frac{108 + 116 + 112}{3} = \frac{336}{3} = 112 \text{ units}$$

Part A is centred upon 2004 and part B on 2008. Plot points 107 and 112 against their middle years, 2004 and 2008. By joining these points, we obtain the required trend line as shown in Fig. 16.6. The line can be extended and used for prediction.

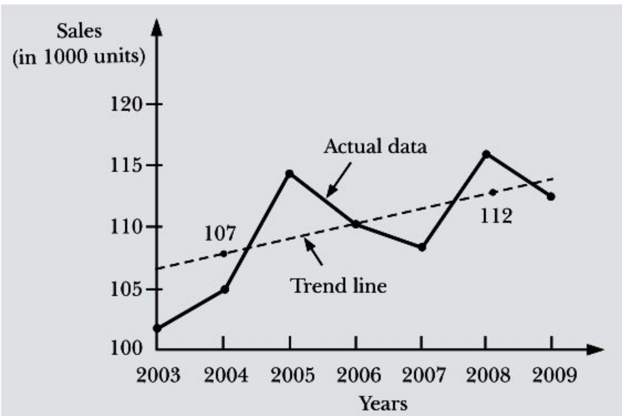


Figure 16.6
Trend Line by the Method of Semi-average

To calculate the time-series $\hat{y} = a + bx$, we need

$$\text{Slope} = b = \frac{\Delta y}{\Delta x} = \frac{\text{change in sales}}{\text{change in year}}$$

$$= \frac{112 - 107}{2008 - 2004} = \frac{5}{4} = 1.25$$

$$\text{Intercept} = a = 107 \text{ units at } 2004$$

Thus, the trend line is $\hat{y} = 107 + 1.25x$

Since 2012 is 8-year distant from the origin (2004), therefore we have

$$\hat{y} = 107 + 1.25(8) = 117$$

16.8.3 Exponential Smoothing Method

Exponential smoothing method compares *past data from previous time periods with exponentially decreasing importance in the forecast so that the most recent data carries more weight in the moving average*. Simple exponential smoothing makes no explicit adjustment for trend effects whereas adjusted exponential smoothing does take trend effects into account (see the next section for details).

Exponential Smoothing Method: A quantitative forecasting method that uses a weighted average of past time-series values to arrive at new time-series values.

Simple Exponential Smoothing

Simple exponential smoothing method is helpful in **forecasting** the value for the present time period X_t multiplied by an exponential smoothing constant α (not the same as used for Type I error) falling between 0 and 1 plus the product of the present time period forecast F_t and $(1 - \alpha)$. Mathematically, it is written as

$$F_{t+1} = \alpha X_t + (1 - \alpha) F_t = F_t + \alpha(X_t - F_t) \quad (16\text{-}1a)$$

where F_{t+1} is the forecast for the next time period $(t + 1)$; F_t is the forecast for the present time period (t); α is a weight called exponentially smoothing constant $(0 \le \alpha \le 1)$ and X_t is the actual value for the present time period (t).

If exponential smoothing is used over long period of time, then forecast for F_t is expressed as

$$F_t = \alpha X_{t-1} + (1 - \alpha) F_{t-1}. \quad (16\text{-}1b)$$

More weight is given to past data when *smoothing constant* α is low, and more weight is given to recent data when it is high. For example, if $\alpha = 0.9$, then 99.99 per cent of the forecast value is determined by the four most recent periods. But if $\alpha = 0.1$, then only 34.39 per cent of the forecast value is determined by these last 4 periods and the smoothing effect is equivalent to a 19-period arithmetic moving average.

If $\alpha = 1$, then each forecast would reflect total adjustment to the recent period value and the forecast would simply be last period's actual value, i.e., $F_t = 1.0D_{t-1}$. Since fluctuations are random, the value of α is generally kept in the range of 0.005 to 0.30 in order to 'smooth' the forecast.

The following table illustrates forecast value in different time periods. For example, when $\alpha = 0.5$, the new forecast is based on the values in the last three or four periods. When $\alpha = 0.1$, a very small weight is given to recent values and takes a 19-period arithmetic moving average.

Forecast: A projection or prediction of future values of a time-series.

Smoothing Constant	*Weight Assigned to*				
	Most Recent Period (α)	*2nd Most Recent Period* $\alpha(1-\alpha)$	*3rd Most Recent Period* $\alpha(1-\alpha)^2$	*4th Most Recent Period* $\alpha(1-\alpha)^3$	*5th Most Recent Period* $\alpha(1-\alpha)^4$
$\alpha = 0.1$	0.1	0.09	0.081	0.073	0.066
$\alpha = 0.5$	0.5	0.25	0.125	0.063	0.031

Selecting the Smoothing Constant: To get an accurate forecast, it is important assign an appropriate value to the exponential smoothing constant, α.

The most appropriate value of α which is equal to an arithmetic moving average, in terms of degree of smoothing, can be estimated as $\alpha = 2/(n + 1)$. Lower the difference between forecasted values and the actual or observed values, the accuracy of a forecasting model is judged more.

Error: The error of an individual forecasting model is defined as

$$\text{Forecast error} = \text{Actual values} - \text{Forecasted values}$$

$$e_t = X_t - F_t$$

The *mean absolute deviation (MAD) method* is used to measure the forecast error for a forecasting model. The MAD is computed by taking the sum of the absolute values of the individual forecast errors and then dividing by number of periods n as follows:

$$\text{MAD} = \frac{\sum|\text{Forecast errors}|}{n}$$

where standard deviation $\sigma \cong 1.25$ MAD

The exponential smoothing method facilitates continuous updating of the estimate of MAD. The current MADt is given by

$$\text{MAD}t = \alpha\ |\ \text{Actual values} - \text{Forecasted values}\ |\ + (1 - \alpha)\ \text{MAD}_{t-1}$$

Higher values of smoothing constant, α make the current MAD more responsive to current forecast errors.

Example 16.7: A firm uses simple exponential smoothing with $\alpha = 0.1$ to forecast demand. The forecast for the week of February 01 was 500 units whereas actual demand turned out to be 450 units.

(a) Forecast the demand for the week of February 08.
(b) Assume the actual demand during the week of February 08 turned out to be 505 units. Forecast the demand for the week of February 15. Continue forecasting through March 15, assuming that subsequent demands were actually 516, 488, 467, 554, and 510 units.

Solution: Given $F_{t-1} = 500$, $D_{t-1} = 450$, and $\alpha = 0.1$

(a) $F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1}) = 500 + 0.1(450 - 500) = 495$ units
(b) Forecast of demand for the week of February 15 is shown in Table 16.5.

Table 16.5 Forecast of Demand

Week	*Demand* D_{t-1}	*Old Forecast* F_{t-1}	*Forecast Error* $(D_{t-1} - F_{t-1})$	*Correction* $\alpha(D_{t-1} - F_{t-1})$	*New Forecast* (F_t) $F_{t-1} + \alpha(D_{t-1} - F_{t-1})$
Feb. 1	450	500	−50	−5	495
8	505	495	10	1	496
15	516	496	20	2	498
22	488	498	−10	−1	497
Mar. 1	467	497	−30	−3	494
8	554	494	60	6	500
15	510	500	10	1	501

If no previous forecast value is known, the old forecast starting point may be estimated or taken to be an average of some preceding periods.

Example 16.8: A hospital has used a 9-month moving average forecasting method to predict drug and surgical inventory requirements. The actual demand for one item is shown in the table below. Using the previous moving average data, convert to an exponential smoothing forecast for month 33.

Month :	24	25	26	27	28	29	30	31	32
Demand :	78	65	90	71	80	101	84	60	73

Solution: The moving average of a 9-month period is given by

$$\text{Moving average} = \frac{\sum \text{Demand } (x)}{\text{Number of periods}} = \frac{78 + 65 + \ldots + 73}{9} = 78$$

Assume $F_{t-1} = 78$. Therefore, estimated $\alpha = \frac{2}{n+1} = \frac{2}{9+1} = 0.2$

Thus, $F_t = F_{t-1} + \alpha (D_{t-1} - F_{t-1}) = 78 + 0.2(73 - 78) = 77$ units

Adjusted Exponential Smoothing

In simple exponential smoothing model, the smoothing effect can be increased or decreased by increasing or decreasing the value of α. However, if a trend exists in the data, then for an increasing trend the forecasts will be consistently low and for decreasing trends it will be consistently high. Simple exponential smoothing forecasts may be adjusted $(F_t)_{adj}$ for trend effects by adding a trend smoothing factor β to the forecasted value, F_t.

$$(F_t)_{adj} = F_t + \frac{1-\beta}{\beta} T_t$$

where $(F_t)_{adj}$ is trend-adjusted forecast; F_t is simple exponential smoothing forecast; β is smoothing constant for trend and T_t is exponentially smoothed trend factor.

Values of β can be found by the trial-and-error approach. The high value of the trend smoothing constant β gives more weight to recent changes in trend and a low β gives less weight to the most recent trends that tends to smooth out the present trend.

The value of the exponentially smoothed trend factor (T_t) is computed as follows:

$$T_t = \beta(F_t - F_{t-1}) + (1 - \beta)T_{t-1}$$

where T_{t-1} is the last period trend factor.

Simple exponential smoothing is often referred to as *first-order smoothing* and trend-adjusted smoothing is referred to as *second-order* or *double smoothing*.

Example 16.9: Develop an adjusted exponential forecast for the firm in Example 16.7. Assume the initial trend adjustment factor (T_{t-1}) is zero and $\beta = 0.1$.

Solution: Table 16.6 presents information needed to develop an adjusted exponential forecast.

Table 16.6

Week	D_{t-1}	F_{t-1}	F_t
Feb. 1	450	500	495
8	505	495	496
15	516	496	498
22	488	498	497
Mar. 1	467	497	494
8	554	494	500
15	510	500	501

The trend adjustment is an addition of a smoothing factor $\{(1 - \beta)/\beta\}T_t$ to the simple exponential forecast, so we need the previously calculated forecast values. Letting the first $T_{t-1} = 0$, we have

Week 2/1 : $T_t = \beta (F_t - F_{t-1}) + (1 - \beta)T_{t-1}$

$= 0.1(495 - 500) + (1 - 0.1)(0) = -0.50$

Adjusted forecast, $(F_t)_{adj} = F_t + \frac{1-\beta}{\beta} T_t = 495 + \frac{1-0.1}{0.1}(-0.50) = 490.50$

Week 2/8 : $T_t = 0.1(496 - 495) + 0.9(-0.50) = -0.35$

Adjusted forecast, $(F_t)_{adj} = 496 + 9(-0.35) = 492.85$

Putting the remainder of the calculations in table form, the trend-adjusted forecast for the week of March 15 is $(F_t)_{adj}$ = 501.44 compared to the simple exponential forecast of F_t = 500, which is not a large difference.

Self-practice Problems 16A

16.1 The owner of a small company manufactures a product. Since he started the company, the number of units of the product he has sold is represented by the following time-series:

Year	: 2005	2006	2007	2008	2009	2010	2011
Units sold	: 100	120	95	105	108	102	112

Find the trend line that describes the trend by using the method of semi-averages.

16.2 Fit a trend line to the following data by the freehand method:

Year	*Production of Steel (million tonnes)*	*Year*	*Production of Steel (million tonnes)*
2000	20	2005	25
2001	22	2006	23
2002	24	2007	26
2003	21	2008	25
2004	23		

16.3 A State Govt. is studying the number of traffic fatalities in the state resulting from drunken driving for each of the last 12 months:

Month	*Accidents*
1	280
2	300
3	280
4	280
5	270
6	240
7	230
8	230
9	220
10	200
11	210
12	200

Find the trend line that describes the trend by using the method of semi-averages.

16.4 Calculate the three-month moving averages from the following data:

Jan.	Feb.	March	April	May	June
57	65	63	72	69	78
July	Aug.	Sept.	Oct.	Nov.	Dec.
82	81	90	92	95	97

[*Osmania Univ., B.Com., 2006*]

16.5 Gross revenue data (₹ in million) for a Travel Agency for a 10-year period is as follows:

Year	*Revenue*
2001	3
2002	6
2003	10
2004	8
2005	7
2006	12
2007	14
2008	14
2009	18
2010	19

Calculate a 3-year moving average for the revenue earned.

16.6 The owner of small manufacturing company has been concerned about the increase in manufacturing costs over the past 10 years. The following data provide a time-series of the cost per unit for the company's leading product over the past 10 years.

Year	*Cost per Unit*	*Year*	*Cost per Unit*
2000	332	2005	405
2001	317	2006	410
2002	357	2007	427
2003	392	2008	405
2004	402	2009	438

Calculate a 5-year moving average for the unit cost of the product.

16.7 The following data provide a time-series of the number of commercial and industrial unit failures during the period 1994–2009.

Year	*No. of Failures*	*Year*	*No. of Failures*
1994	23	2002	9
1995	26	2003	13
1996	28	2004	11
1997	32	2005	14
1998	20	2006	12
1999	12	2007	9
2000	12	2008	3
2001	10	2009	1

Calculate a 5-year and 7-year moving average for the number of units failure.

16.8 Estimate the trend values using the data given by taking a four-year moving average:

Year	*Value*	*Year*	*Value*
1990	12	1997	100
1991	25	1998	82
1992	39	1999	65
1993	54	2000	49
1994	70	2001	34
1995	87	2002	20
1996	105	2003	7

[*Madras Univ., M.Com., 2008*]

16.9 In January, a city hotel predicted a February demand for 142-room occupancy. Actual February demand was 153 rooms. Using a smoothing constant of $\alpha = 0.20$, forecast the March demand using the exponential smoothing model.

16.10 A shoe manufacturer, using exponential smoothing with $\alpha = 0.1$, has developed a January trend forecast of 400 units for a ladies' shoe. This brand has seasonal indexes of 0.80, 0.90, and 1.20 respectively for the first three months of the year. Assuming actual sales were 344 units in January and 414 units in February, what would be the seasonalized March forecast?

16.11 A food processor uses exponential smoothing (with $\alpha = 0.10$) to forecast next month's demand. Past (actual) demand in units and the simple exponential forecasts up to month 51 are shown in the following table

Month	*Actual Demand*	*Old Forecast*
43	105	100.00
44	106	100.50
45	110	101.05
46	110	101.95
47	114	102.46
48	121	103.61
49	130	105.35
50	128	107.82
51	137	109.84

(a) Using simple exponential smoothing, forecast the demand for month 52.

(b) Suppose a firm wishes to start including a trend-adjustment factor of $\beta = 0.60$. If it assumes an initial trend adjustment of zero ($T_t = 0$) in month 50, what would be the value of $(F_t)_{adj}$ for month 52?

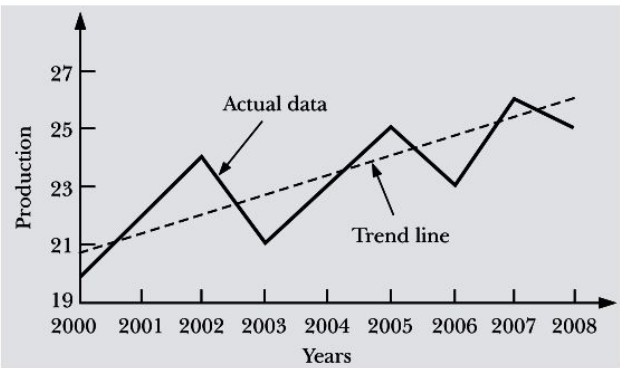
Hints and Answers

16.1

Year (y)	*Units Sold* (x)	
2005	100	
2006	120	315/3 = 105.00 = a
2007	95	
2008	105	
2008	108	
2010	102	322/3 = 107.33 = b
2011	112	

Trend line $y = 105 + 107.33x$.

16.2



16.3

Month	*Accidents*
1	280
2	300
3	280
4	280
5	270
6	240
7	230
8	230
9	220
10	200
11	210
12	200

Average of first 6 months, $a = 1650/6 = 275$
Average of last 6 months, $b = 1290/6 = 215$
Trend line $y = 275 + 215x$.

16.4

Month	*Values*	*3-month Total*	*3-month Moving Average*
Jan.	57	—	—
Feb.	65	185	185/3 = 61.67
March	63	200	200/3 = 66.67
April	72	204	204/3 = 68.00
May	69	219	73.00
June	78	229	76.33
July	82	241	80.33
Aug.	81	253	84.33
Sept.	90	263	87.67
Oct.	92	277	92.38
Nov.	95	284	94.67
Dec.	97	—	—

16.5

Year	*Revenue*	*3-year Moving Total*	*3-year Moving Average*
2001	3	—	
2002	6	19	19/3 = 6.33
2003	10	24	24/3 = 8.00
2004	8	21	21/3 = 7.00
2005	7	25	8.33
2006	12	32	10.66
2007	14	34	11.33
2008	14	46	15.33
2009	18	51	17.00
2010	19	—	—

16.6

Year	*Per Unit Cost*	*5-year Moving Total*	*5-year Moving Average*
2000	332	—	—
2001	317	—	—
2002	357	1800	1800/5 = 360.0
2003	392	1873	1873/5 = 374.6
2004	402	1966	1966/5 = 393.2
2005	405	2036	407.2
2006	410	2049	409.8
2007	427	2085	417.0
2008	405	—	—
2009	438	—	—

16.7

Year	*Number of Failures*	*5-year Moving Total*	*5-year Moving Average*	*7-year Moving Total*	*7-year Moving Average*
1994	23	—	—	—	—
1995	26	—	—	—	—
1996	28	129	25.8	—	—
1997	32	118	23.6	153	21.9
1998	20	104	20.8	140	20.0
1999	12	86	17.2	123	17.6
2000	12	63	12.6	108	15.4
2001	10	56	11.2	87	12.4
2002	9	55	11.0	81	11.6
2003	13	57	11.4	81	11.6
2004	11	59	11.8	78	11.1
2005	14	59	11.8	71	10.1
2006	12	69	9.8	63	5.0
2007	9	39	7.9	—	—
2008	3	—	—	—	—
2009	1	—	—	—	—

16.8

Year	*Value*	*4-year Moving*	*4-year Moving*	*4-year Average Total*	*Moving Centred Average*
1990	12	—	—	—	—
1991	25	—	—	—	—
		130	130/4=32.5		
1992	39				(32.5 + 47)/2 = 39.75
		188	188/4=47.0		
1993	54				(47 + 62.5)/2 = 54.75
		250	250/4=62.5		
1994	70				70.75
		316	79.0		
1995	87				84.75
		362	90.5		
1996	105				92.00
		374	93.5		
1997	100				90.75
		352	88.0		
1998	82				81.00
		296	74.0		
1999	65				65.75
		230	57.5		
2000	49				49.75
		168	42.0		
2001	34				34.75
		110	27.5		
2002	20	—	—		—
2003	7	—	—		—

16.9 New forecast (March demand)

$= F_{t-1} + \alpha\,(D_{t-1} - F_{t-1})$

$= 142 + 0.20\,(153 - 142) = 144.20 = 144$ rooms

16.10 (a) Deseasonalized actual January demand

$= 344/0.80 = 430$ units

(b) Compute the deseasonalized forecast

$F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})$

$= 400 + 0.1\,(430 - 400) = 403$

16.11 In this problem the smoothing constant for the original data ($\alpha = 0.10$) differs from the smoothing constant for the trend $\beta = 0.60$.

(*a*) $F_t = F_{t-1} + \alpha\,(D_{t-1} - F_{t-1})$

$= 109.84 + 0.1(137.00 - 109.84)$

$= 112.56$

(*b*) Forecast for month 51:

$$(F_t)_{adj} = F_t + \frac{1-\beta}{\beta} T_t$$

where $T_t = \beta\,(F_t - F_{t-1}) + (1-\beta)T_{t-1}$

$= 0.6\,(109.84 - 107.82) + (1 - 0.6)$

$= 1.21$

$$(F_t)_{adj} = 109.84 + \left(\frac{1-0.6}{0.6}\right)(1.21)$$

$= 110.65$

Forecast for month 52:

$T_t = \beta(F_t - F_{t-1}) + (1-\beta)T_{t-1}$

$= 0.6\,(112.56 - 109.84) + (1 - 0.6)\,(1.21)$

$= 2.12$

$$(F_t)_{adj} = F_t + \frac{1-\beta}{\beta} T_t$$

$$= 112.56 + \left(\frac{1-0.6}{0.6}\right)(2.12) = 113.98$$

16.9 TREND PROJECTION METHODS

A *trend* best represented by a straight line is termed as long-run direction (upward, downward or constant) of any business activity over a period of several years.

The trend projection method fits a trend line to a time-series data and then projects medium-to-long-range forecasts. Several trend projection methods such as exponential and quadratic can be used to fit trend line depending upon movement of time-series data. In this section, linear, quadratic and exponential trend models are discussed. Since seasonal variations can influence trend analysis, it is assumed that no such variations effect the data or are removed before establishing the trend.

Reasons to Study Trend: A few reasons to study trends are as follows:

(i) The study of trend helps to describe the long-term general direction of any business activity over a long period of time.

(ii) The study of trend facilitate in making intermediate and long-term forecasting projections.

16.9.1 Linear Trend Projection Method

The ***method of least squares*** from regression analysis is used to find the ***trend line of best fit*** to a time-series data. The trend line of best fit is defined by the following equation:

$$\hat{y} = a + bx$$

where $\hat{y}$ is the predicted value of the dependent variable; a is the y-axis intercept; b is the slope of the regression line (or the rate of change in y for a given change in x); and x is the independent variable (which is *time* in this case).

Characteristics of Trend Line of Best Fit

(i) The sum of all vertical deviations about the line of best fit is zero, i.e., $\sum(y - \hat{y}) = 0$.

(ii) The sum of all vertical deviations squared is minimum, i.e., $\sum(y - \hat{y})$ is least.

(iii) The line of best fit passes through the mean values of variables x and y.

The value of two constants a and b can be found by the simultaneous solution of normal equations:

$$\sum y = na + b\sum x \quad \text{and} \quad \sum xy = a\sum x + b\sum x^2$$

Alternately, values of constants a and b for any regression lines are obtained as follows:

$$b = \frac{\sum x\,y - n\,\overline{x}\,\overline{y}}{\sum x^2 - n(\overline{x})^2} \quad \text{and} \quad a = \overline{y} - b\overline{x}$$

Example 16.10: Below are given the figures of production (in thousand quintals) of a sugar factory:

Year	:	2000	2001	2002	2003	2004	2005	2006
Production	:	80	90	92	83	94	99	92

(a) Fit a straight line trend to these figures.
(b) Plot these figures on a graph and show the trend line.
(c) Estimate the production in 2010. [*Bangalore Univ., B.Com., 2008*]

Solution: (a) Using normal equations and the sugar production data, we can compute constants a and b as shown in Table 16.7:

Table 16.7 Calculation for Least-squares Equation

Year	*Time Period (x)*	*Production (x)*	x^2	*xy*	*Trend Values* $\hat{y}$
2000	1	80	1	80	84
2001	2	90	4	180	86
2002	3	92	9	276	88
2003	4	83	16	332	90
2004	5	94	25	470	92
2005	6	99	36	594	94
2006	7	92	49	644	96
	28	630	140	2576	

$$\overline{x} = \frac{\Sigma x}{n} = \frac{28}{7} = 4,\ \overline{y} = \frac{\Sigma y}{n} = \frac{630}{7} = 90$$

$$b = \frac{\Sigma x\,y - n\,\overline{x}\,\overline{y}}{\Sigma x^2 - n(\overline{x})^2} = \frac{2576 - 7(4)(90)}{140 - 7(4)^2} = \frac{56}{28} = 2$$

$$a = \overline{y} - b\,\overline{x} = 90 - 2(10) = 70$$

Therefore, linear trend component for the production of sugar is

$$\hat{y} = a + bx = 82 + 2x$$

The slope $b = 2$ indicates that over the past 7 years, the production of sugar had an average growth of about 2 thousand quintals per year.

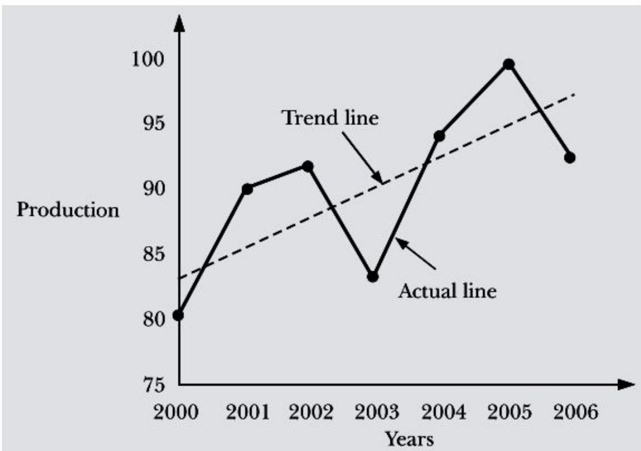


Figure 16.7
Linear Trend for Production of Sugar

(b) Plotting points on the graph paper, we get an actual graph representing production of sugar over the past 7 years. Join the point $a = 82$ and $b = 2$ (corresponds to 1996) on the graph we get a trend line as shown in Fig. 16.7.
(c) The production of sugar for year 2010 will be

$$\hat{y} = 82 + 2\,(10) = 102 \text{ thousand quintals}$$

Example 16.11: The following table relates to the tourist arrivals (in millions) during 1994 to 2000 in India:

Year :	2001	2002	2003	2004	2005	2006	2007
Tourists arrivals :	18	20	23	25	24	28	30

Fit a straight line trend by the method of least squares and estimate the number of tourists that would arrive in the year 2011. *[Kurukshetra Univ., MTM, 2007]*

Solution: Using normal equations and tourists arrival data we can compute constants a and b as shown in Table 16.8:

Table 16.8 Calculations for Least-Squares Equation

Year	*Time Scale* (x)	*Tourist Arrivals* y	xy	x^2
2001	−3	18	−54	9
2002	−2	20	−40	4
2003	−1	23	−23	1
2004	0	25	0	0
2005	1	24	24	1
2006	2	28	56	4
2007	3	30	90	9
		168	53	28

$$\bar{x} = \frac{\sum x}{n} = 0,\ \bar{y} = \frac{\sum y}{n} = \frac{168}{7} = 24$$

$$b = \frac{\sum xy - n\bar{x}\,\bar{y}}{\sum x^2 - n(\bar{x})^2} = \frac{53}{28} = 1.893;$$

$$a = \bar{y} - b\bar{x} = 24 - 1.893(0) = 24$$

Therefore, the linear trend component for arrival of tourists is

$$\hat{y} = a + bx = 24 + 1.893x$$

The estimated number of tourists that would arrive in the year 2011 are

$$\hat{y} = 24 + 1.893\,(10) = 5.07 \text{ million (measured from 2001 = origin)}$$

16.9.2 Quadratic Trend Projection Method

The quadratic trend line (also called the *parabola)* for estimating value of a dependent variable y from an independent variable x is expressed as

$$\hat{y} = a + bx + cx^2$$

To find the estimated value of the variable y, the values of constants a, b and c in such a trend line can be determined by solving following three normal equations:

$$\sum y = na + b\sum x + c\sum x^2$$
$$\sum xy = a\sum x + b\sum x^2 + c\sum x^3$$
$$\sum x^2 y = a\sum x^2 + b\sum x^3 + c\sum x^4$$

The values of these constants can also be calculated by using the following shortest method:

$$a = \frac{\sum y - c\sum x^2}{n};\ b = \frac{\sum xy}{\sum x^2};\ \text{and } c = \frac{n\sum x^2 y - \sum x^2 \sum y}{n\Sigma x^4 - \left(\sum x^2\right)^2}$$

Example 16.12: The prices of a commodity during 2005–2010 are given below. Fit a parabola to these data. Estimate the price of the commodity for the year 2004.

Year	*Price*	*Year*	*Price*
2005	100	2008	140
2006	107	2009	181
2007	128	2010	192

Also plot the actual and trend values on a graph.

Solution: To fit a quadratic trend line, $\hat{y} = a + bx + cx^2$, calculations to compute values of constants a, b and c are shown in Table 16.9.

Table 16.9 Calculations for Parabola Trend Line

Year	*Time Scale (x)*	*Price (y)*	x^2	x^3	x^4	xy	x^2y	*Trend Values* ($\hat{y}$)
2005	−2	100	4	−8	16	−200	400	97.72
2006	−1	107	1	−1	1	−107	107	110.34
2007	0	128	0	0	0	0	0	126.68
2008	1	140	1	1	1	140	140	146.50
2009	2	181	4	8	16	362	724	169.88
2010	3	192	9	27	81	576	1728	196.82
	3	848	19	27	115	771	3099	847.94

(i) $\sum y = na + b\sum x + c\sum x^2$ or $848 = 6a + 3b + 19c$

(ii) $\sum xy = a\sum x + b\sum x^2 + c\sum x^3$ or $771 = 3a + 19b + 27c$

(iii) $\sum x^2y = a\sum x^2 + b\sum x^3 + c\sum x^4$ or $3099 = 19a + 27b + 115c$

Eliminating constant a from eqns. (i) and (ii), we get

(iv) $694 = 35b + 35c$

Eliminating constant a from eqns. (ii) and (iii), we get

(v) $5352 = 280b + 168c$

Solving eqns. (iv) and (v) for b and c, we get $b = 18.04$ and $c = 1.78$. Substituting values of b and c in eqn. (i), we get $a = 126.68$. Hence, the required quadratic trend line becomes

$$y = 126.68 + 18.04x + 1.78x^2$$

Substituting, $x = -2, -1, 0, 1, 2,$ and 3 in the trend line plotting these values of x and y on a graph paper as shown in Fig. 16.8.

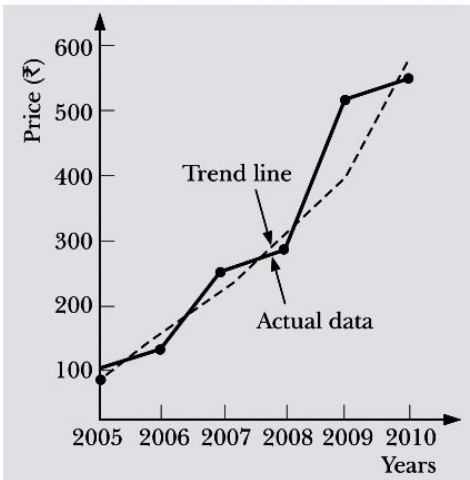


Figure 16.8 Trend Line for Price of Commodity

16.9.3 Exponential Trend Projection Method

If given values of dependent variable y form a geometric progression and values of corresponding independent variable x form an arithmetic progression, then relationship between variables x and y is given by an exponential function, and the best-fitted curve is referred to as *exponential trend*. For example, growth of bacteria, money accrued at compound interest, sales or earnings over a short period and so on follows exponential growth.

An exponential function showing the rate of growth (or change) in the value of dependent variable y with respect to an independent variable x is expressed as

$$y = a\, b^{cx},\ a > 0$$

The letter b is a fixed constant, usually either 10 or e, where a is a constant to be determined.

Taking logarithms (with base 10) of both sides of the above equation, we get

$$\log y = \log a + (c \log b)x \qquad (16\text{-a})$$

For b=10, $\log b = 1$, but for $b = e$, $\log b = 0.4343$ (approx.). In either case, this equation is of the form

$$y' = c + dx \qquad (16\text{-b})$$

where $y' = \log y$, $c = \log a$ and $d = c \log b$.

Equation (16-2) represents a straight line. A method of fitting an exponential trend line to a set of observed values of variable y is equivalent to fit a straight trend line to the logarithm of values of variable y.

In order to compute the values of constants a and b in the exponential function, the following two normal equations are solved:

$$\sum \log y = n \log a + \log b\Sigma x$$
$$\sum x \log y = \log a\Sigma x + \log b\Sigma x^2$$

When $\Sigma x = 0$, the two normal equations become

$$\sum \log y = n \log a \ \text{ or } \ \log a = \frac{1}{n}\sum \log y$$

and

$$\sum x \log y = \log b\sum x^2 \ \text{ or } \ \log b = \frac{\sum x \log y}{\sum x^2}$$

Setting the centre of the time period as $x = 0$, and keep equal number of plus and minus period on each side which sum to zero.

Example 16.13: The sales (₹ in million) of a company for the years 2000 to 2004 are

Year :	2000	2001	2002	2003	2004
Sales :	1.6	4.5	13.8	40.2	125.0

Find the exponential trend for the given data and estimate the sales for 2007.

Solution: The computational time can be reduced by coding the data. For this consider $u = x - 3$. The necessary computations are shown in Table 16.10.

Table 16.10 Calculation for Least-Squares Equation

Year	*Time Period* x	$u = x - 3$	u^2	*Sales* y	*Log y*	*u Log y*
2000	1	–2	4	1.60	0.2041	–0.4082
2001	2	–1	1	4.50	0.6532	–0.6532
2002	3	0	0	13.80	1.1390	0
2003	4	1	1	40.20	1.6042	1.6042
2004	5	2	4	125.00	2.0969	4.1938
			10		5.6983	4.7366

$$\log a = \frac{1}{n}\sum \log y = \frac{1}{5}\,(5.6983) = 1.1397$$

$$\log b = \frac{\sum u \log y}{\sum u^2} = \frac{4.7366}{10} = 0.4737$$

Therefore, $\log y = \log a + (x + 3) \log b = 1.1397 + 0.4737x$. For sales during 2007, $x = 3$, and we obtain

$$\log y = 1.1397 + 0.4737\,(3) = 2.5608$$

or

$$y = \text{antilog}\,(2.5608) = 363.80$$

16.9.4 Changing the Origin and the Scale of Trend Line

While calculating a moving average, the forecasted value is assumed to be centred in the middle of the month or the year. However, the reference point (origin) can be shifted or the units of variables x and y are changed to monthly or quarterly values. The procedure of changing the origin and the scale of trend line is as follows:

(i) Shift the origin by adding or subtracting the desired number of periods from independent variable x in the original trend equation.
(ii) Change the time units from annual values to monthly values by dividing independent variable x by 12.
(iii) Change units of variable y from annual to monthly values by dividing right-hand side of the equation by 12.

Example 16.14: The following forecasting equation has been derived by a least-squares method:

$$\hat{y} = 10.27 + 1.65x \text{ (Base year: 2007; } x = \text{years; } y = \text{tonnes /year)}$$

Rewrite the equation by

(a) shifting the origin to 2012.
(b) expressing x units in months, retaining y in tonnes/year.
(c) expressing x units in months and y in tonnes/month.

Solution: (a) Shift the origin by adding the desired number of period 5 (2007 to 2012) to x in the given equation:

$$\hat{y} = 10.27 + 1.65\,(x + 5) = 18.52 + 1.65x$$

where $2012 = 0$, x = years, y = tonnes/year.
(b) Expressing x units in months

$$\hat{y} = 10.27 + \frac{1.65x}{12} = 10.27 + 0.14x$$

where July 01, 2007 = 0, x = months, y = tonnes/year.
(c) Expressing y in tonnes/month, retaining x in months

$$\hat{y} = \frac{1}{12}(10.27 + 0.14x) = 0.86 + 0.01x$$

where July 01, 2007 = 0, x = months, y = tonnes/month.

Remarks

1. If both x and y are to be expressed in months together, then divide constant 'a' by 12 and constant 'b' by 24 because data are sums of 12 months. Thus, monthly trend equation becomes

Linear trend : $\hat{y} = \dfrac{a}{12} + \dfrac{b}{24}x$

Parabolic trend : $\hat{y} = \dfrac{a}{12} + \dfrac{b}{144}x + \dfrac{c}{1728}x^2$

But if data are given as monthly averages per year, then value of 'a' remains unchanged, 'b' is divided by 12 and 'c' by 144.

2. The annual trend equation can be reduced to quarterly trend equation as

$$\hat{y} = \frac{a}{4} + \frac{b}{4 \times 12}x = \frac{a}{4} + \frac{b}{48}x$$

Self-practice Problems 16B

16.12 The general manager of a building materials production plant feels that the demand for plasterboard shipments may be related to the number of construction permits issued in the country during the previous quarter. The manager has collected the data shown in the table.

Construction Permits	*Plasterboard Shipments*
15	6
9	4
40	16
20	6
25	13
25	9
15	10
35	16

(a) Use the normal equations to derive a regression forecasting equation.
(b) Determine a point estimate for plasterboard shipments when the number of construction permits is 30.

16.13 A company that manufactures steel observed the production of steel (in metric tons) represented by the time-series:

Year	2001	2002	2003	2004	2005	2006	2007
Production of steel	60	72	75	65	80	85	95

(a) Find the linear equation that describes the trend in the production of steel by the company.
(b) Estimate the production of steel in 2008.

16.14 Fit a straight line trend by the method of least squares to the following data. Assuming that the same rate of change continues, what would be the predicted earning (₹ in lakh) for the year 2008?

Year	1999	2000	2001	2002	2003	2004	2005	2006
Earnings	38	40	65	72	69	60	87	95

[*Agra Univ., B.Com., 2006; MD Univ., B.Com., 2008*]

16.15 The sales (₹ in lakh) of a company for the years 1998 to 2004 are given below:

Year	1998	1999	2000	2001	2002	2003	2004
Sales	32	47	65	88	132	190	275

Find trend values by using the equation $yc = a\ bx$ and estimate the value for 2005.

[*Delhi Univ., B.Com., 2006*]

16.16 A company that specializes in the production of petrol filters has recorded the following production (in 1000 units) over the last 7 years.

Years	2001	02	03	04	05	06	07
Production	42	49	62	75	92	122	158

(a) Develop a second-degree estimating equation that best describes these data.
(b) Estimate the production in 2009.

16.17 In 1996 a firm began downsizing in order to reduce its costs. One of the results of these cost cutting measures has been a decline in the percentage of private industry jobs that are managerial. The following data show the percentage of females who are managers from 1996 to 2003.

Years	1996	97	98	99	00	01	02	03
Percentage	6.7	5.3	4.3	6.1	5.6	7.9	5.8	6.1

(a) Develop a linear trend line for this time-series through 2001 only.
(b) Use this trend to estimate the percentage of females who are managers in 2004.

16.18 A company develops, markets, manufactures, and sells integrated wide-area network access products. The following are annual sales (₹ in million) data from 1998 to 2004.

Year	1998	1999	2000	2001	2002	2003	2004
Sales	16	17	25	28	32	43	50

(a) Develop the second-degree estimating equation that best describes these data.
(b) Use the trend equation to forecast sales for 2005.

Hints and Answers

16.12 (a)

x	y	xy	x^2	y^2
15	6	90	225	36
9	4	36	81	16
40	16	640	1,600	256
20	6	120	400	36
25	13	325	625	169
25	9	225	625	81
15	10	150	225	100
35	16	560	1,225	256
184	80	2,146	5,006	950

$n = 8$ pairs of observations;

$$\bar{x} = 184/8 = 23;\ \bar{y} = 80/8 = 10$$

$\sum y = na + b\sum x$ or $80 = 8a + 184b$

$\sum xy = \Sigma x + b\sum x^2$ or $2{,}146 = 184a + 5{,}006b$

After solving equations we get $a = 0.91$ and $b = 0.395$.

Therefore the equation is: $\hat{y} = 0.91 + 0.395x$

(b) For $x = 30$, we have $\hat{y} = 0.91 + 0.395\ (30) = 13$ shipments (approx.)

16.13 $a = \sum y/n = 532/7 = 76$; $b = \sum xy/\sum x^2 = 136/28 = 4.857$

(a) Trend line $\hat{y} = a + bx = 76 + 4.857x$

(b) For 2008, $x = 7$, $\hat{y} = 76 + 4.857(7) = 42.01$ metric tonnes.

16.14 $a = \sum y/n = 526/8 = 65.75$;

$b = \sum xy/\sum x^2 = 616/168 = 3.667$

Trend line: $\hat{y} = a + bx = 65.75 + 3.667x$

For 2008, $x = 11$; $\hat{y} = 65.75 + 3.667\ (11)$

$= ₹\ 106.087$ lakh.

16.15 $\log a = \dfrac{1}{n}\sum \log y = \dfrac{1}{7}(13.7926) = 1.9704$

$$\log b = \frac{\sum x \log y}{\sum x^2} = \frac{4.3237}{28} = 0.154$$

Thus $\log y = \log a + x \log b = 1.9704 + 0.154x$

For 2005, $x = 4$; $\log y = 1.9704 + 0.154\ (4) = 2.5864$

$y = \text{Antilog}\ (2.5864) = ₹385.9$ lakh.

16.16

Year	Period	Deviation from 1998 (x)	x^2	x^4	y	xy	x^2y
2001	1	−3	9	81	42	−126	378
2002	2	−2	4	16	49	−98	196
2003	3	−1	1	1	62	−62	62
2004	4	0	0	0	75	0	0
2005	5	1	1	1	92	+92	92
2006	6	2	4	16	122	+244	488
2007	7	3	9	81	158	+474	1422
		0	28	196	600	524	2638

(a) Solving the equations

$\sum y = na + c\sum x^2$ or $600 = 7a + 28c$

$\sum x^2 y = a\sum x^2 + c\sum x^4$ or $2638 = 28a + 196c$

$\sum xy = b\sum x^2$ or $524 = 28b$

We get $a = 80.05$, $b = 18.71$ and $c = -1.417$

Hence $\hat{y} = a + bx + cx^2 = 80.05 + 18.71x - 1.417x^2$

(b) For 2009, $x = 8$; $\hat{y} = 80.05 + 18.71(8) - 1.417(8)^2$

$= ₹\ 139.042$ thousand.

16.17

Year	Time Period	Deviation from 2001 x	Percentage of Females y	xy	x^2
1996	1	−5	6.7	−33.5	25
1997	2	−4	5.3	−21.2	16
1998	3	−3	4.3	−12.9	9
1999	4	−2	6.1	−12.2	4
2000	5	−1	5.6	−6.6	1
2001	6	0	7.9	0	0
2002	7	1	5.8	5.8	1
2003	8	2	6.1	12.2	4
		−12	47.8	−68.4	60

(a) Solving the equations

$\sum y = na + b\sum x$ or $47.8 = 8a - 12b$

$\sum xy = a\sum x + b\sum x^2$ or $-67.4 = -12a + 60b$

We get $a = 6.28$ and $b = 0.102$

Hence $\hat{y} = a + bx = 6.128 + 0.102x$

(b) For 2004, $x = 9$; $\hat{y} = 6.128 + 0.102(9)$

$= 7.046$ per cent.

16.18

Year	Time Period	Deviation from 2001 (x)	Sales y	xy	x^2	x^4	x^2y
1998	1	−3	16	−48	9	81	144
1999	2	−2	17	−34	4	16	68
2000	3	−1	25	−25	1	1	25
2001	4	0	28	0	0	0	0
2002	5	1	32	32	1	1	32
2003	6	2	43	86	4	16	172
2004	7	3	50	150	9	81	450
		0	211	161	28	196	891

(a) Solving the equations

$\sum y = na + c\sum x^2$ or $211 = 7a + 28c$

$\sum x^2 y = a\sum x^2 + c\sum x^4$ or $891 = 28a + 196c$

$\sum xy = b\sum x^2$ or $161 = 28b$

We get $a = 27.904$, $b = 5.75$ and $c = 0.559$

$\hat{y} = a + bx + cx^2 = 27.904 + 5.75x + 0.559x^2$

For 2008, $x = 8$; $\hat{y} = 27.904 + 5.75(8) + 0.559\ (8)^2$

$= 82.848$

16.10 MEASUREMENT OF SEASONAL EFFECTS

Seasonal effect is defined as the repetitive and predictable pattern of data behaviour in a time-series around the trend line during particular time intervals of the year. In order to measure (or detect) the seasonal effect, time period must be less than one year such as days, weeks, months or quarters.

Seasonal influences arise due to (i) natural changes in the seasons during the year, (ii) habits, customs, or (iii) festivals that occur at the same time each year. There are three main reasons to study seasonal effects:

(i) The description of the seasonal effect provides a better understanding of the impact of seasonal component of time-series on a particular phenomenon.

(ii) The seasonal influence (if any) can be eliminated from the time-series in order to observe the effect of the other components of time-series such as cyclical and irregular. The process of eliminating the effects of seasonality from a time-series data is referred to as ***deseasonalization*** or ***seasonal adjustment*** of the data.

(iii) The knowledge of seasonal influences or effects on time-series data is essential for short-term forecast.

Deseasonalization: A statistical process used to remove the effect of seasonality from a time-series by dividing each original series observation by the corresponding seasonal index.

Seasonal influences are removed from a time-series data by dividing the actual *y* value for each quarter by its corresponding seasonal index:

$$\text{Deseasonalized value} = \frac{\text{Actual quarterly value}}{\text{Seasonal index of corresponding quarter}} \times 100$$

The deseasonalized values of variable *y* are measured in the same unit as the actual values, and reflect the collective influence of *trend*, *cyclical* and *irregular* components.

Remarks:

1. In an additive time-series model, the seasonal component may be estimated by using following relationship:

$$S = Y - (T + C + I)$$

In the absence of C and I, S = Y – T, i.e. Seasonal component = Actual data values – trend values.

2. In a multiplicative time-series model, the seasonal component may be isolated by decomposition. The decomposition process begins by first computing TC and then dividing the time-series data (T.C.S.I) by TC. The resulting expression contains seasonal effects along with irregular fluctuations:

$$\frac{\text{T.C.S.I}}{\text{T.C}} = \text{S.I.}$$

A method for eliminating irregular fluctuations can be applied, leaving only the seasonal effects as shown below:

$$\text{Seasonal effect} = \frac{\text{T.S.C.I}}{\text{T.C.I}} = \frac{\text{Y}}{\text{T.C.I}} \times 100\%$$

3. The time-series data can be deseasonalized by dividing the actual values Y by adjusted seasonal effects as follows:

$$\frac{Y}{S} = \frac{\text{T.S.C.I}}{S} = \text{T. C. I} \times 100\% \quad \leftarrow \text{Multiplicative}$$

$$Y - S = (T + S + C + I) - S = T + C + I \quad \leftarrow \text{Additive Model}$$

16.10.1 Seasonal Index

Seasonal effects are measured in terms of an index called *seasonal index* attached to each period of the time-series within a year. Hence, if monthly data are considered, there are 12 seasonal indexes, one for each month. Similarly for quarterly data, there are 4 indexes. *A seasonal index is an average that indicates the percentage deviation of actual values of the time-series from a base value which excludes the short-term seasonal influences.*

Each adjusted seasonal index measures the average magnitude of seasonal influence on the actual values of the time-series for a given period within a year. By subtracting the base index of 100 (which represents the T and C components) from each seasonal index, the magnitude of seasonal influence can be measured.

The following four methods are used to construct seasonal indexes to measure seasonal influence on time-series data:

1. Method of simple averages
2. Ratio-to-trend method
3. Ratio-to-moving average method
4. Link relatives method.

16.10.2 Method of Simple Averages

Since ***simple average method*** also called ***average percentage method*** expresses the data of each month or quarter as a percentage of the average of the year. The steps of the method are summarized below:

(i) Average the unadjusted data by years, months or quarters.

(ii) Add the figures of each month and obtain the averages by dividing the monthly totals by the number of years. Let the averages for 12 months be $\bar{x}_1$, $\bar{x}_2$,..., $\bar{x}_3$.

(iii) Obtain an average of monthly averages by dividing the total of monthly averages by 12:

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_{12}}{12}$$

(iv) Express monthly averages as percentages of the grand average to compute seasonal index of each month as follows:

$$\text{Seasonal index for month, } i = \frac{\text{Monthly average for month } i}{\text{Average of monthly averages}} \times 100$$

$$= \frac{\bar{x}_i}{\bar{\bar{x}}} \times 100 \ (i = 1, 2, \ldots, 12)$$

The average of the indexes should always be 100, i.e., sum of the indexes should be 1200 for 12 months, and sum should be 400 for 4 quarterly data. But, if this sum is not 1200, then the monthly percentage are adjusted by multiplying these by a suitable factor [1200 ÷ (sum of the 12 values)].

Example 16.15: The seasonal indexes of the sale of readymade garments in a store are given below:

Quarter	*Seasonal Index*
January to March	98
April to June	90
July to September	82
October to December	130

If the total sales of garments in the first quarter is worth ₹ 1,00,000, determine how much worth of garments of this type should be kept in stock to meet the demand in each of the remaining quarters. *[Delhi Univ., B.Com., 2006]*

Solution: Calculations of seasonal index for each quarter and estimated stock (in ₹) is shown in Table 16.11.

Table 16.11 Calculation of Estimated Stock

Quarter	*Seasonal Index (SI)*	*Estimated Stock (₹)*
Jan. —March	98	1,00,000.00
April—June	90	91,836.73*
July —Sept.	82	83,673.45
Oct. —Dec.	130	1,32,653.06

* These figures are calculated as follows:

$$\text{Seasonal index for second quarter} = \frac{\text{Figure for first quarter} \times \text{SI for second quarter}}{\text{SI for first quarter}}$$

$$\text{Seasonal index for third quarter} = \frac{\text{Figure for first quarter} \times \text{SI for third quarter}}{\text{SI for first quarter}}$$

Example 16.16: Use the method of monthly averages to determine the monthly indexes for the data of production of a commodity for the years 2008 to 2010.

Month	*2008*	*2009*	*2010*
January	15	23	25
February	16	22	25
March	18	28	35
April	18	27	36
May	23	31	36
June	23	28	30
July	20	22	30
August	28	28	34
September	29	32	38
October	33	37	47
November	33	34	41
December	38	44	53

Solution: Computation of seasonal index by average percentage method based on the data is shown in Table 16.12.

Table 16.12 Calculation of Seasonal Indexes

Month	*2008*	*2009*	*2010*	*Monthly Total for 3 Years*	*Monthly Averages for 3 Years*	*Percentage Average of Monthly Averages*
Jan.	15	23	25	63	21	70
Feb.	16	22	25	63	21	70
March	18	28	35	81	27	90
April	18	27	36	81	27	90
May	23	31	36	90	30	100
June	23	28	30	81	27	90
July	20	22	30	72	24	80
Aug.	28	28	34	90	30	100
Sept.	29	32	38	99	33	110
Oct.	33	37	47	117	39	130
Nov.	33	34	41	108	36	120
Dec.	38	44	53	135	45	150
				1080	360	1200

Monthly Average: 1080/12 = 90; 360/12 = 30; 1200/12 = 100

The average of monthly averages is obtained by dividing the total of monthly averages by 12. In column 7, each monthly average for 3 years has been expressed as a percentage of the averages. For example, the percentage for January is

Monthly index for January = (21/3) × 100 = 70
February = (21/3) × 100 = 70
March = (27/3) × 100 = 90, and so on

Example 16.17: The data on prices (₹ in per kg) of a certain commodity during 2007 to 2011 are shown below:

Quarter	*Years*				
	2007	*2008*	*2009*	*2010*	*2011*
I	45	48	49	52	60
II	54	56	63	65	70
III	72	63	70	75	84
IV	60	56	65	72	66

Compute the seasonal indexes by the average percentage method and obtain the deseasonalized values.

Solution: Calculations for quarterly averages are shown in Table 16.13.

Table 16.13 Calculation Seasonal Indexes

Year	*Quarters*			
	I	*II*	*III*	*IV*
2007	45	54	72	60
2008	48	56	63	56
2009	49	63	70	65
2010	52	65	75	72
2011	60	70	84	66
Quarterly total	254	308	364	319
Quarterly average	50.8	61.6	72.8	63.8
Seasonal index	81.60	98.95	116.94	102.48

$$\text{Average of quarterly averages} = \frac{50.8 + 61.6 + 72.8 + 63.8}{4} = \frac{249}{4} = 62.25$$

$$\text{Thus, Seasonal index for quarter I} = \frac{50.8}{62.25} \times 100 = 81.60$$

$$\text{Seasonal index for quarter II} = \frac{61.6}{62.25} \times 100 = 98.95$$

$$\text{Seasonal index for quarter III} = \frac{72.8}{62.25} \times 100 = 116.94$$

$$\text{Seasonal index for quarter IV} = \frac{63.8}{62.25} \times 100 = 102.48$$

Seasonal influences are removed from a time-series data by dividing the actual y value for each quarter by its corresponding seasonal index. The deseasonalized values are given in Table 16.14.

$$\text{Deseasonalized value} = \frac{\text{Actual quarterly value}}{\text{Seasonal index of corresponding quarter}} \times 100$$

Table 16.14 Calculation for Least-Squares Equation

Year	*Quarters*			
	I	*II*	*III*	*IV*
2007	55.14	54.57	61.57	58.54
2008	58.82	56.59	53.87	54.64
2009	60.00	63.66	59.85	63.42
2010	63.72	65.68	64.13	70.25
2011	73.52	70.74	71.83	64.40

Limitations of simple averages method

This method assumes that either there is no trend component in the series, that is, C × S × I = 0 or trend has little impact on the time-series.

16.10.3 Ratio-to-Trend Method

Ratio-to-trend method also known as ***percentage trend method*** assumes that seasonal variation for a given month is a constant fraction of trend. This method isolates the seasonal effect using following equation:

$$\frac{T \cdot S \cdot C \cdot I}{T} = S \times C \times I$$

The steps of the method are summarized as follows:

1. Applying the least-squares method to compute the trend values.
2. Eliminate the trend value using a multiplicative model.
3. Arrange the percentage data values obtained in Step (ii) according to months or quarters as the case may be for various years.
4. Find the monthly (or quarterly) averages arranged in Step (iii) using a suitable measures of central tendency—arithmetic mean and median.
5. Find the grand average of monthly averages found in Step (iv). If the grand average is 100, then the monthly averages represent seasonal indexes. Otherwise, an adjustment is made by multiplying each index by a suitable factor [1200/(sum of the 12 values)] to get the final seasonal indexes.

Example 16.18: Quarterly sales data (₹ in million) in a super bazar are presented in the following table.

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	60	80	72	68
2001	68	104	100	88
2002	80	116	108	96
2003	108	152	136	124
2004	160	184	172	164

Calculate the seasonal index for each of the four quarters using the ratio-to-trend method.

Solution: Calculations to obtain annual trend values from the given quarterly data using the method of least-squares are shown in Table 16.15.

Table 16.15 Calculation of Trend Values

Year	*Yearly Total (1)*	*Yearly Average* $y = (2)/4$	*Deviation from Mid-Year* x	x^2	xy	*Trend Values* $\hat{y}$
2000	280	70	−2	4	−140	64
2001	360	90	−1	1	−90	88
2002	400	100	0	0	0	0
2003	520	130	1	1	130	112
2004	680	170	2	4	340	160
		560		10	240	

Solving the following normal equations, we get

$$\Sigma y = na + b\Sigma x \quad \text{or} \quad 560 = 5a, \quad \text{i.e., } a = 112$$

$$\Sigma xy = a\Sigma x + b\Sigma x_2 \quad \text{or} \quad 240 = 10b, \quad \text{i.e., } b = 24$$

Thus, the yearly fitted trend line is $y = 112 + 24x$. The value of $b = 24$ indicates yearly increase in sales. Thus, the quarterly increment will be 24/4 = 6.

To calculate quarterly trend values, consider first the year 2000. The trend value for this year is 64. This is the value for the middle of the year 2000, that is, half of the 2nd quarter and half of the 3rd quarter. Since quarterly increment is 6, the trend value for the 2nd quarter of 2000 would be 64 – (6/2) = 61 and for the 3rd quarter it would be 64 + (6/2) = 67. The value for the 1st quarter of 2000 would be 61 – 6 = 55 and for the 4th quarter it would be 67 + 6 = 73. Similarly, trend values of the various quarters of other years can be calculated as shown in Table 16.16.

Table 16.16 Quarterly Trend Values

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	55	61	67	73
2001	79	85	91	97
2002	103	109	115	121
2003	127	133	139	145
2004	151	157	163	169

After getting the trend values, the given data values in the time-series are expressed as percentages of the corresponding trend values in Table 16.16. Thus for the 1st quarter of 2000, this percentage would be (60/55) × 100 = 109.09; for the 2nd quarter it would be (80/61) × 100 = 131.15, and so on. Other values can be calculated in the same manner as shown in Table 16.17.

Table 16.17 Ratio-to-Trend Values

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	109.09	131.15	107.46	93.15
2001	86.08	122.35	109.89	90.72
2002	77.67	106.42	93.91	79.34
2003	85.04	114.29	97.84	85.52
2004	105.96	117.20	105.52	97.04
Total	463.84	591.41	514.62	445.77
Average	92.77	118.28	102.92	89.15
Adjusted				= 403.12
seasonal index	92.02	117.33	102.09	88.43

The total of average of seasonal indexes is 403.12 (>400). Thus, we apply the correction factor (400/403.12) = 0.992. Now each quarterly average is multiplied by 0.992 to get the adjusted seasonal index as shown in Table 16.17.

The seasonal index 92.02 in the first quarter means that on average sales trend to be depressed by the presence of seasonal forces to the extent of approx. (100 – 92.02) = 7.98 per cent. Alternatively, values of time-series would be approx. (7.98/92.02)×100 = 8.67 per cent higher had seasonal influences not been present.

16.10.4 Ratio-to-Moving Average Method

This method is also called the percentage ***moving average method***. In this method, the original values in the time-series data are expressed as percentages of moving averages instead of percentages of trend values in the ratio-to-trend method. The steps of the method are summarized as follows:

(i) Find the centred 12 monthly (or 4 quarterly) moving averages of the original data values in the time-series.
(ii) Express each original data value of the time-series as a percentage of the corresponding centred moving average values obtained in Step (i). In other words, in a multiplicative time-series model, we get

$$\frac{\text{Original data values}}{\text{Trend values}} \times 100 = \frac{T \cdot C \cdot S \cdot I}{T \cdot C} \times 100 = (S \times I) \times 100\%$$

(iii) This implies that the ratio-to-moving average represents the seasonal and irregular components.
(iv) Arrange these percentages according to months or quarters of given years. Find the averages over all months or quarters of the given years.
(v) If the sum of these indexes is not 1200 (or 400 for quarterly figures), multiply them by a correction factor = 1200/ (sum of monthly indexes). Otherwise, the 12 monthly averages will be considered as seasonal indexes.

Example 16.19: Calculate the seasonal index by the ratio-to-moving method from the following data:

Year	*Quarters*			
	I	*II*	*III*	*IV*
2009	75	60	53	59
2010	86	65	63	80
2011	90	72	66	85
2012	100	78	72	93

Solution: Calculations for 4 quarterly moving averages and ratio-to-moving averages are shown in Table 16.18.

Table 16.18 Calculation of Ratio-to-Moving Averages

Year	*Quarter*	*Original Values* $Y = T.C.S.I$	*4-Quarter Moving Total*	*4-Quarter Moving Average*	*2× 4-Quarter Moving Average T.C*	*Ratio-to-Moving Average (Per cent)* $\frac{Y}{T.C} = (S.I)100\%$
2009	1	75	—	—	—	—
	2	60	248		—	—
	3	54	259	507	63.375	54/63.375 = 85.20
	4	59	264	523	65.375	59/65.375 = 90.25
2010	1	86	273	537	67.125	128.12
	2	65	294	567	70.875	91.71
	3	62	298	592	74.000	85.13
	4	80	305	603	75.375	106.14
2011	1	90	308	613	76.625	117.43
	2	72	313	521	77.625	92.75
	3	66	323	636	79.500	83.02
	4	85	329	652	81.500	104.29
2012	1	100	335	664	84.750	92.03
	2	78	343	678	84.750	92.03
	3	72	—	—	—	—
	4	93	—	—	—	—

Table 16.19 Calculation of Seasonal Index

Year	Quarters			
	I	II	III	IV
2009	—	—	85.21	90.25
2010	128.12	91.71	85.13	106.14
2011	117.45	92.75	85.13	104.29
2012	120.48	92.03	—	—
Total	366.05	276.49	255.47	300.68
Seasonal average	91.51	69.13	63.87	75.17 = 299.66
Adjusted seasonal index	122.07	92.22	85.20	100.30 ≅ 400

The total of seasonal averages is 299.66. Therefore, the corresponding correction factor would be 400/299.68 = 1.334. Each seasonal average is multiplied by the correction factor 1.334 to get the adjusted seasonal indexes shown in Table 16.19.

Example 16.20: Calculate the seasonal indexes by the ratio-to-moving average method from the following data:

Year	*Quarter*	*Actual Values (Y = T.C.S.I)*	*4-quarterly Moving Average*	*Year*	*Quarter*	*Given Values (Y)*	*4-quarterly Moving Average*
2007	1	75	—	2009	1	90	76.625
	2	60	—		2	72	77.625
	3	54	63.375		3	66	79.500
	4	59	65.375		4	85	81.500
2008	1	86	67.125	2010	1	100	83.000
	2	65	70.875		2	78	84.750
	3	63	74.000		3	72	—
	4	80	75.375		4	93	—

Solution: Calculations of ratio-to-moving averages are shown in Table 16.20.

Table 16.20 Calculation of Seasonal Indexes

Year	*Quarter*	*Actual Values (Y = T.C.S.I)*	*4-quarterly Moving (T.C)*	*Ratio to Moving Average (Percentage)* $\frac{Y}{T.C} \times 100$
2007	1	75	—	—
	2	60	—	—
	3	54	63.375	85.21
	4	59	65.375	90.25
2008	1	86	67.125	128.12
	2	65	70.875	91.71
	3	63	74.000	85.14
	4	80	75.375	106.14
2009	1	90	76.625	117.46
	2	72	77.625	92.75
	3	66	79.500	83.02
	4	85	81.500	104.29
2010	1	100	83.000	120.84
	2	78	84.750	92.04
	3	72	—	—
	4	93	—	—

Rearranging the percentages to moving averages, the seasonal indexes are calculated as shown in Table 16.21.

Table 16.21 Seasonal Indexes

Year	*Quarter (Percentages to Moving Averages)*			
	1	*2*	*3*	*4*
2007	—	—	85.21	90.25
2008	128.12	91.71	85.14	106.14
2009	117.46	92.75	83.02	104.30
2010	120.48	92.04	—	—
Total	366.06	276.50	253.37	300.69
Average	122.02	92.17	84.46	100.23 = 398.88
Adjusted seasonal index	$\frac{122.02}{99.72}\times 100$	$\frac{92.17}{99.72}\times 100$	$\frac{84.46}{99.72}\times 100$	$\frac{100.23}{99.72}\times 100$
	= 122.36	= 92.43	= 84.70	= 100.51 = 400

Since the total of average indexes is less than 400, the adjustment of the seasonal index has been done by calculating the grand mean value as follows:

$$\bar{\bar{x}} = \frac{122.02 + 92.17 + 84.46 + 100.23}{4} = 99.72$$

The seasonal average values are now converted into adjusted seasonal indexes using $\bar{\bar{x}} = 99.72$ as shown in Table 16.21.

Advantages and Disadvantages of Ratio-to-Moving Average Method

This method eliminates both trend and cyclical variations from the time-series. However, if cyclical variations are not regular, then this method is not capable of eliminating them completely. Seasonal indexes calculated by this method will contain some effect of cyclical variations.

The only disadvantage of this method is that six values at the beginning and the six values at the end are not taken into consideration for calculation of seasonal indexes.

16.10.5 Link Relative Method

This method is also known as ***Pearson's method***. The percentages (also ***link relatives***) obtained by this method link each month to its preceding month. The steps of this method are summarized below:

1. Convert the monthly (or quarterly) data into link relatives by using the formula:

$$\text{Link relative for a month} = \frac{\text{Data value of current month}}{\text{Data value of preceding month}} \times 100$$

2. Calculate the average of link relatives of each month using either median or arithmetic mean.
3. Convert the link relatives (L.R.) into chain relatives (C.R.) by using the formula:

$$\text{C.R. for a month} = \frac{[\text{L.R. of current month (or quarter)} \times \text{C.R. of preceding month (or quarter)}]}{100}$$

The C.R. for the first month (or quarter) is assumed to be 100.

4. Compute the new chain relative for January (first month) on the basis of December (last month) using the formula:

$$\text{New C.R. for January} = \frac{\text{C.R. of January} \times \text{C.R. of December}}{100}$$

Generally, new C.R. is not equal to 100 and therefore needs to be multiplied with the monthly correction factor:

$$d = \frac{1}{12}(\text{New C.R. for January} - 100)$$

If quarterly data is given, then the correction factor would be

$$d = \frac{1}{4}(\text{New C.R. of first quarter} - 100)$$

The corrected C.R. for any month can be calculated by using the formula:

Corrected C.R. for kth month = Original C.R. of kth month – $(k - 1)\,d$

where $k = 1, 2, 3,..., 12$

5. Find the mean of the corrected chain index. If it is 100, then the corrected chain indexes represent the seasonal variation indexes. Otherwise divide the corrected C.R. of each month (or quarter) by the mean value of corrected C.R. and then multiply by 100 to get the seasonal variation indexes.

Example 16.21: Apply the method of link relatives to the following data and calculate seasonal indexes.

Year	*Quarters*			
	I	*II*	*III*	*IV*
2008	68	62	61	63
2009	65	58	56	61
2010	68	63	63	67
2011	70	59	56	62
2012	60	55	51	58

Solution: Computations of link relatives (L.R.) are shown in Table 16.22 by using the following formula:

$$\text{Link relative of any quarter} = \frac{\text{Data value of current quarter}}{\text{Data value of preceeding quarter}} \times 100$$

Table 16.22 Computation of Link Relatives

Year	*Quarters*			
	I	*II*	*III*	*IV*
2008	—	91.18	98.39	103.28
2009	103.18	89.23	96.55	108.93
2010	111.48	92.65	100.00	106.35
2011	104.48	84.29	94.91	110.71
2012	96.78	91.67	92.73	113.73
Total of L.R.	415.92	449.02	482.58	543.00
Arithmetic mean of L.R.	103.98	89.80	96.52	108.60
Chain relatives (C.R.)	100	$\frac{89.80 \times 100}{100}$ = 89.80	$\frac{96.52 \times 89.80}{100}$ = 86.67	$\frac{108.60 \times 86.67}{100}$ = 94.12

The new chain relatives for the first quarter on the basis of last quarter are as follows:

$$\text{New C.R.} = \frac{\text{L.R. of first quarter} \times \text{C.R. of previous quarter}}{100}$$

$$= \frac{103.98 \times 94.12}{100} = 97.9$$

Since new C.R. is not equal to 100, therefore applying quarterly correction factor:

$$d = \frac{1}{4}\ (\text{New C.R. of first quarter} - 100) = \frac{1}{4}(97.9 - 100) = -0.53$$

Thus, the corrected (or adjusted) C.R. for other quarters computed by using the following formula is shown in Table 16.23:

Corrected C.R. for kth quarter = Original C.R. of kth quarter – $(k - 1)d$

where $k = 1, 2, 3, 4$.

Table 16.23 Calculation of Link Relatives

Quarter	*I*	*II*	*III*	*IV*
Corrected C.R.	100	89.80 – (–0.53) = 90.33	86.67 – 2(–0.53) = 87.73	94.13 – 3(–0.53) = 95.71
Seasonal indexes	$\frac{100}{93.44} \times 100$ = 107.02	$\frac{90.33}{93.44} \times 100$ = 96.67	$\frac{87.73}{93.44} \times 100$ = 93.89	$\frac{95.71}{93.44} \times 100$ = 102.42

$$\text{Mean of corrected C.R.} = \frac{100 + 90.33 + 87.73 + 95.71}{4} = 93.44$$

$$\text{Seasonal variation index} = \frac{\text{Corrected C.R.}}{\text{Mean of corrected C.R.}} \times 100$$

Example 16.22: Apply the method of link relatives to the following data and calculate the seasonal index:

Year	*Quarters*			
	I	*II*	*III*	*IV*
2008	45	54	72	60
2009	48	56	63	56
2010	49	63	70	65
2011	52	65	75	72
2012	60	70	84	86

Solution: Computations of link relatives (L.R.) using the following formula are shown in Table 16.24.

$$\text{L.R. of any quarter} = \frac{\text{Data value of current quarter}}{\text{Data value of preceding quarter}} \times 100$$

Table 16.24 Computation of Link Relatives

Year	*Quarters*			
	I	*II*	*III*	*IV*
2008	—	120	133.33	83.33
2009	80.00	116.67	112.50	88.89
2010	87.50	128.57	111.11	92.86
2011	80.00	125.00	115.38	96.00
2012	85.71	116.67	120.00	78.57
Total of L.R.	333.21	606.91	592.32	439.65
Arithmetic mean of L.R.	83.30	121.38	118.46	87.93
Chain relatives (C.R.)	100	$\frac{121.38 \times 100}{100}$ = 121.38	$\frac{118.46 \times 121.38}{100}$ = 143.78	$\frac{87.93 \times 143.78}{100}$ = 126.42

The new chain relatives for the first quarter on the basis of the preceding quarter are calculated as follows:

$$\text{New C.R.} = \frac{\text{L.R. of first quarter} \times \text{C.R. of previous quarter}}{100}$$

$$= \frac{83.30 \times 126.42}{100} = 105.30$$

Since the new C.R. is more than 100, therefore we need to apply a quarterly correction factor as:

$$d = \frac{1}{4} \text{ (New C.R. of first quarter – 100)}$$

$$= \frac{1}{4} (105.30 - 100) = 1.325$$

Thus the corrected (or adjusted) C.R. for other quarters computed by using the following formula is shown in Table 16.25.

$$\text{Corrected C.R. for } k\text{th quarter} = \text{Original C.R. of } k\text{th quarter} - (k - 1)\, d$$

where $k = 1, 2, 3, 4$.

Table 16.25 Corrected C.R.

Quarters	*I*	*II*	*III*	*IV*
Corrected C.R.	100	121.38 – 1.32 = 120.06	143.78 – 2(1.32) = 141.14	126.42 – 3(1.32) = 122.46
Seasonal indexes	$\frac{100}{120.92} \times 100$ = 82.70	$\frac{120.06}{120.92} \times 100$ = 99.30	$\frac{141.14}{120.92} \times 100$ = 116.72	$\frac{122.46}{120.92} \times 100$ = 101.27

$$\text{Mean of corrected C.R.} = \frac{100 + 120.06 + 141.14 + 122.46}{4} = 120.92$$

$$\text{Seasonal variation index} = \frac{\text{Corrected C.R.}}{\text{Mean of corrected C.R.}} \times 100$$

Advantages and Disadvantages of Link Relative Method This method is simpler than the ratio-to-trend or the ratio-to-moving average methods. The L.R. of the first quarter (or month) is not taken into consideration as compared to ratio-to-trend method, where 6 values each at the beginning and at the end periods are lost.

This method eliminates the trend provided there is a straight line (linear) trend in the time-series—which is generally not formed in business and economic.

16.11 MEASUREMENT OF CYCLICAL VARIATIONS—RESIDUAL METHOD

In a multiplicative time-series model, the four components of time-series: secular trend (T), seasonal variation (S), cyclical variation (C) and irregular variation (I) are written as

$$y = \text{T. C. S. I}$$

The deseasonalized data can be adjusted for trend analysis by dividing original data, y by the corresponding trend and seasonal variation values. This leaves behind cyclical (C) and irregular (I) variations in the data set as shown below:

$$\frac{y}{\text{T.S}} = \frac{\text{T.C.S.I}}{\text{T.S}} = \text{C. I}$$

Again with the use of moving averages of suitable period, the effect of irregular variations may be eliminated or reduced leaving behind only the cyclical variations. Recall that cyclical variations in time-series tend to oscillate above and below the secular trend line for periods longer than one year.

The procedure of identifying cyclical variation is known as the ***residual method***. The steps of residual method are summarized as follows:

1. Obtain seasonal indexes and deseasonalized data.
2. Obtain trend values and expressed seasonalized data as percentages of the trend values.
3. Divide the original data (y) by the corresponding trend values (T) in the time-series to get S. C. I. Further divide S. C. I. by S to get C. I.
4. Smooth out irregular variations by using moving averages of desired period but of short duration, leaving only the cyclical variation.

16.12 MEASUREMENT OF IRREGULAR VARIATIONS

Since irregular variations are random in nature, no particular procedure can be followed to isolate and identify these variations. However, the residual method can be extended to do the needful by dividing C. I. by cyclical component (C) to identify the irregular component (I).

Alternately, trend (T), seasonal (S) and cyclical (C) components of the given time-series are estimated and then the residual is taken as the irregular variation. Thus, in the case of multiplicative time-series model, we have

$$\frac{Y}{T.C.S} = \frac{T.C.S.I}{T.C.S} = I$$

where S and C are in fractional form and not in percentages.

Conceptual Questions 16B

15. (a) Under what circumstances can a trend equation be used to forecast a value in a series in the future? Explain.
(b) What are the advantages and disadvantages of trend analysis? When would you use this method of forecasting?

16. What effect does seasonal variability have on a time-series? What is the basis for this variability for an economic time-series?

17. What is measured by a moving average? Why are 4-quarter and 12-month moving averages used to develop a seasonal index?

18. Briefly describe the moving average and least squares methods of measuring trend in time-series.

19. Explain the simple average method of calculating indexes in the context of time-series analysis.

20. Distinguish between ratio-to-trend and ratio-to-moving average as methods of measuring seasonal variations. Which is better and why?

21. Distinguish between trend, seasonal variations, and cyclical variations in a time-series. How can trend be isolated from variations?

22. Describe any two important methods of trend measurement, and examine critically the merits and demerits of these methods.

23. Why do we deseasonalize data? Explain the ratio-to-moving average method to compute the seasonal index.

24. Explain the following:
(a) '..., the business analyst who uses moving averages to smoothen data, while in the process of trying to discover business cycles, is likely to come up with some non-existent cycles'.
(b) 'Despite great limitations of statistical forecasting, the forecasting techniques are invaluable to the economist, the businessman, and the Government.'

25. 'A 12-month moving average of time-series data removes trend and cycle'. Do you agree ? Why or why not?

26. Why do we deseasonalize data? Explain the ratio-to-moving average method to compute the seasonal index.

27. Explain the methods of fitting of the quadratic and exponential curves. How would you use the fitted curves for forecasting?

28. 'A key assumption in the classical method of time-series analysis is that each of the component movements in the time-series can be isolated individually from a series'. Do you agree with this statement? Does this assumption create any limitation to such analysis?

Self-practice Problems 16C

16.19 Apply the method of link relatives to the following data and calculate seasonal indexes.

Quarter	*2001*	*2002*	*2003*	*2004*	*2005*
I	6.0	5.4	6.8	7.2	6.6
II	6.5	7.9	6.5	5.8	7.3
III	7.8	8.4	9.3	7.5	8.0
IV	8.7	7.3	6.4	8.5	7.1

16.20 A company estimates its sales for a particular year to be ₹ 24,00,000. The seasonal indexes for sales are as follows:

Month	*Seasonal Index*	*Month*	*Seasonal Index*
January	75	July	102
February	80	August	104
March	98	September	100
April	128	October	102
May	137	November	82
June	119	December	73

Using this information, calculate estimates of monthly sales of the company. (Assume that there is no trend).

[*Osmania Univ., MBA, 2007*]

16.21 Calculate the seasonal index from the following data using the average method:

Year	*Quarter*			
	I	*II*	*III*	*IV*
2000	72	68	80	70
2001	76	70	82	74
2002	74	66	84	80
2003	76	74	84	78
2004	78	74	86	82

[*Kerala Univ., B.Com., 2006*]

16.22 Calculate seasonal index numbers from the following data:

Year	*Quarter*			
	I	*II*	*III*	*IV*
2002	108	130	107	93
2003	86	120	110	91
2004	92	118	104	88
2005	78	100	94	78
2006	82	110	98	86
2007	106	118	105	98

16.23 Calculate seasonal index for the following data by using the average method:

Year	*Quarters*			
	I	*II*	*III*	*IV*
2000	72	68	80	70
2001	76	70	82	74
2002	74	66	84	80
2003	76	74	84	78
2004	78	74	86	82

16.24 On the basis of quarterly sales (₹ in lakh) of a certain commodity for the years 2003–2004, the following calculations were made:

Trend: $y = 20 + 0.5t$ with origin at first quarter of 2003

where t = time unit (one quarter),

y = quarterly sales (₹ in lakh)

Seasonal variations:

Quarter :	1	2	3	4
Seasonal index :	80	90	120	110

Estimate the quarterly sale for the year 2003 using multiplicative model.

Hints and Answers

16.19

Year	Quarters			
	I	II	III	IV
2001	—	108.3	120.0	111.5
2002	62.1	146.3	106.3	89.9
2003	93.2	95.6	143.1	68.8
2004	112.5	80.6	129.3	113.3
2005	77.6	110.6	109.6	88.8
Arithmetic average	$\frac{345.4}{4} = 86.35$	$\frac{541.4}{5} = 108.28$	$\frac{608.3}{5} = 121.66$	$\frac{469.3}{5} = 93.86$
Chain relatives	100	$\frac{100 \times 108.28}{100}$ = 108.28	$\frac{121.66 \times 108.28}{100}$ = 131.73	$\frac{93.86 \times 131.73}{100}$ = 123.65
Corrected chain relatives	100	108 – 1.675 = 106.325	131.73 – 3.35 = 128.38	123.64 – 5.025 = 118.615
Seasonal indexes	$\frac{100 \times 100}{113.4}$ = 88.18	$\frac{106.605}{113.4} \times 100$ = 94.01	$\frac{128.38}{113.4} \times 100$ = 113.21	$\frac{118.615}{113.4} \times 100$ = 104.60

16.20 Seasonal indexes are usually expressed as percentages. The total of all the seasonal indexes is 1200.

Seasonal effect = Seasonal index + 100

The yearly sales being ₹ 24,00,000, the estimated monthly sales for a specified month:

$$\text{Estimated sales} = \frac{\text{Annual sales}}{12} \times \text{Seasonal effect}$$

$$= \frac{24,00,000}{12} \times \text{Seasonal effect}$$

$$= 2,00,000 \times \text{Seasonal effect}$$

Month (*1*)	*Seasonal Index* (*2*)	*Seasonal Effect* (*3*) = (*2*) ÷ *100*	*Estimated Sales* (*4*)=(*3*)×*2,00,000*
January	75	0.75	1,50,000
February	80	0.80	1,60,000
March	98	0.98	1,96,000
April	128	1.28	2,56,000
May	137	1.37	2,74,000
June	119	1.19	2,38,000
July	102	1.02	2,04,000
August	104	1.04	2,08,000
September	100	1.00	2,00,000
October	102	1.02	2,04,000
November	82	0.82	1,64,000
December	73	0.73	1,46,000
	1200	12.00	24,00,000

16.21

Year	Quarterss			
	I	II	III	IV
2000	72	68	80	70
2001	76	70	82	74
2002	74	66	84	80
2003	76	74	84	78
2004	78	74	86	82
Total	376	352	416	384
Average	75.2	70.4	83.2	76.8
Seasonal index	98.43	92.15	108.9	100.52

$$\text{Grand average} = \frac{75.2 + 70.4 + 83.2 + 76.8}{4} = 76.4$$

Seasonal index for quarter

$$k = \frac{\text{Average of quarter } k}{\text{Grand average}} \times 100$$

16.22

Year	Quarters			
	I	II	III	IV
2002	108	130	107	93
2003	86	120	110	91
2004	92	118	104	88
2005	78	100	94	78
2006	82	110	98	86
2007	106	118	105	98
Total	552	696	618	534
Average	92	116	103	89
Seasonal Index	$\frac{92}{100} \times 100$ = 92	$\frac{116}{100} \times 100$ = 116	$\frac{103}{100} \times 100$ = 103	$\frac{89}{100} \times 100$ = 89

Sales in different quarters:

I: ₹ 20,000; II: 20,000×1.16 = ₹ 23,200;

III: 20,000 × 1.03 = ₹ 20,600;

IV: 20,000 × 0.89 = ₹ 17,800

16.23

Year	Quarters			
	I	II	III	IV
2000	72	68	80	70
2001	76	70	82	74
2002	74	66	84	80
2003	76	74	84	78
2004	78	74	86	82
Total	376	352	416	384
Average	75.2	70.4	83.2	76.8
Seasonal	$\frac{75.2}{76.4} \times 100$	$\frac{70.4}{76.4} \times 100$	$\frac{83.2}{76.4} \times 100$	$\frac{76.8}{76.4} \times 100$
Index	= 98.43	= 92.15	= 108.90	= 100.52

16.24

Quarter of 2003	*Time Unit*	*Trend (T) Values* $y = 20 + 0.5t$	*Seasonal Effect or Seasonal Index (S)*	*Estimated Sales (₹ in lakh)* $T \cdot S$
1	4	20 + 0.5 × 4 = 22.0	0.80	17.60
2	5	20 + 0.5 × 5 = 22.5	0.90	20.25
3	6	20 + 0.5 × 6 = 23.0	1.20	27.60
4	7	20 + 0.5 × 7 = 23.5	1.10	25.85

Formulae Used

1. Secular trend line
   - Linear trend model
   $$y = a + bx$$
   where $a = \bar{y} - b\bar{x}$; $b = \dfrac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n(\bar{x})^2}$
   - Exponential trend model
   $$y = ab^x;$$
   $$\log a = \frac{1}{n}\sum \log y;\ \log b = \frac{\sum x \log y}{\sum x^2}$$
   - Parabolic trend model
   $$y = a + bx + cx^2$$
   where $a = \dfrac{\sum y - c\sum x^2}{n}$; $b = \dfrac{\sum xy}{\sum x^2}$
   $$c = \frac{n\sum x^2 y - \sum x^2 \sum y}{n\sum x^4 - \left(\sum x^2\right)^2}$$
2. Moving average
   $$MA_{t+1} = \frac{\sum\{D_t + D_{t-1} + \ldots + D_{t-n+1}\}}{n}$$
   where t = current time period
   D = actual data value
   n = length of time period
3. Simple exponential smoothing
   $$F_t = F_{t-1} + \alpha(D_{t-1} - F_{t-1})$$
   where F_t = current period forecast
   F_{t-1} = previous period forecast
   α = a weight ($0 \le \alpha \le 1$)
   D_{t-1} = previous period actual demand
4. Adjusted exponential smoothing
   $$(F_t)_{adj} = F_t + \frac{1-\beta}{\beta} T_t$$
   Where β = smoothing constant for trend
   T_t = exponential smoothed trend factor

Chapter Concepts Quiz

True or False

1. [T] [F] Exponential smoothing is an example of a causal model.
2. [T] [F] Secular trends represent the long-term direction of a time-series.
3. [T] [F] The repetitive movement around a trend line in a one-year period is best described by seasonal variation.
4. [T] [F] The present of trend should not be used for predicting future cyclical variations.
5. [T] [F] A time-series model incorporates the various factors that might influence the quantity being forecast.

6. [T] [F] In exponential smoothing, when the smoothing constant is high, more weight is placed on the more recent data.

7. [T] [F] Seasonal variation is a repetitive and predictable variation around the trend line within a year.

8. [T] [F] In a trend-adjusted exponential smoothing model, a high value for the trend smoothing constant β implies that we wish to make the model less responsive to recent changes in trend.

9. [T] [F] A time-series should be deseasonalized after the trend or cyclical components of the time-series have been identified.

10. [T] [F] The weakness of causal forecasting methods is that we must first forecast the value of the independent variable.

11. [T] [F] No single forecast methodology is appropriate under all conditions.

12. [T] [F] Regression analysis can only be used to develop a forecast based upon a single independent variable.

13. [T] [F] Exponential smoothing is a weighted moving average model where all previous values are weighted with a set of weights that decline exponentially.

14. [T] [F] Periods of moving averages are determined by the periodicity of the time-series.

15. [T] [F] No trend values are lost when determined by the method of moving averages.

Multiple Choice Questions

16. Forecasting time horizons include
(a) long range (b) medium range
(b) short range (d) all of these

17. A forecast that projects company's sales is the
(a) economic forecast (b) technological forecast
(c) demand forecast (d) none of these

18. Quantitative methods of forecasting include
(a) sales force composite
(b) consumer market survey
(c) smoothing approach
(d) all of these

19. Decomposing a time-series refers to breaking down past data into the components of
(a) constants and variations
(b) trends, cycles and random variations
(c) tactical and operational variations
(d) long-term, short-term and medium-term variations

20. Consider a time-series of data for the quarters of 1995 and 1996. The third quarter of 1996 would be coded as:
(a) 2 (b) 3
(c) 5 (d) 6

21. If a time-series has an even number of years and we use coding, then each coded interval is equal to
(a) one month (b) 6 months
(c) one year (d) two years

22. The cyclical variation in the time-series is eliminated by
(a) second-degree analysis
(b) spearman analysis
(c) relative cyclical residual
(d) none of these

23. Suppose a time-series is fitted with parabolic trend model $\hat{y} = a + bx + cx^2$. What do the x's represent in this model?
(a) coded values of the time variables
(b) variable to be determined
(c) estimates of the dependent variable
(d) none of these

24. A component of time-series used for short-term forecast is
(a) trend (b) seasonal
(c) cyclical (d) irregular

25. In an additive time-series model, the component measurements are
(a) positive (b) negative
(c) absolute (d) none of these

26. After detrending, the time-series multiplicative model is represented as:
(a) Y = T S C I (b) Y = S C I
(c) Y = T S I (d) none of these

27. In a time-series multiplicative model its components S, C, and I have
(a) positive values (b) absolute values
(d) indexing values (d) none of these

28. With regard to a regression-based forecast, the standard error of estimate gives a measure of
(a) overall accuracy of the forecast
(b) time period for which the forecast is valid
(b) maximum error of the forecast
(d) all of these

29. Seasonal indexes are calculated by using
(a) freehand curve method
(b) moving average method
(c) link relative method
(d) none of these

30. Suppose a time-series is described by the equation $\hat{y} = 10 + 2x + 7x^2$ based on data for the years 1994–2000. What is the forecast value of $\hat{y}$ for the year 2001?
(a) 130 (b) 195
(c) 245 (d) 800

Concepts Quiz Answers

1. F	**2.** T	**3.** F	**4.** T	**5.** F	**6.** T	**7.** T	**8.** F	**9.** F
10. T	**11.** T	**12.** F	**13.** T	**14.** T	**15.** F	**16.** (d)	**17.** (c)	**18.** (c)
19. (b)	**20.** (c)	**21.** (b)	**22.** (d)	**23.** (a)	**24.** (b)	**25.** (c)	**26.** (b)	**27.** (d)
28. (a)	**29.** (c)	**30.** (a)						

Review Self-practice Problems

16.25 A sugar mill is committed to accepting beets from local producers and has experienced the following supply pattern (in thousands of tonnes/year and rounded).

Year	*Tonnes*	*Year*	*Tonnes*
2000	100	2005	400
2001	100	2006	400
2002	200	2007	600
2003	600	2008	800
2004	500	2009	800

The operations manager would like to project a trend to determine what facility additions will be required by 2012

(a) Sketch a freehand curve and extend it to 2012. What would be your 2012 forecast based upon the curve?

(b) Compute a three-year moving average and plot it as a dotted line on your graph.

16.26 Use the data of Problem 16.25 and the normal equations to develop a least squares line of best fit. Omit the year 2000.

(a) State the equation when the origin is 2005.

(b) Use your equation to estimate the trend value for 2012.

16.27 A forecasting equation is of the form:

$$\hat{y}_c = 720 + 144x$$

[2003 = 0, x unit = 1 year, y = annual sales]

(a) Forecast the annual sales rate for 2003 and also for one year later.

(b) Change the time (x) scale to months and forecast the annual sales rate at July 1, 2003, and also at one year later.

(c) Change the sales (y) scale to monthly and forecast the monthly sales rate at July 1, 2003, and also at one year later.

16.28 Data collected on the monthly demand for an item were as shown below:

January	100
February	90
March	80
April	150
May	240
June	320
July	300
August	280
September	220

(a) What conclusion can you draw with respect to the length of moving average versus smoothing effect?

(b) Assume that the 12-month moving average centred on July was 231. What is the value of the ratio-to-moving average that would be used in computing a seasonal index?

16.29 The data shown in the table below gives the number of lost-time accidents over the past seven years in a cement factory:

Year	*Number Employees (in '000)*	*Number Accidents*
1996	15	5
1997	12	20
1998	20	15
1999	26	18
2000	35	17
2001	30	30
2002	37	35

(a) Use the normal equations to develop a linear time-series equation forecasting the number of accidents.

(b) Use your equation to forecast the number of accidents in 2005.

16.30 Consider the following time-series data:

Week :	1	2	3	4	5	6
Value :	8	13	15	17	16	9

(a) Develop a 3-week moving average for this time-series. What is the forecast for week 7?

(b) Use $\alpha = 0.2$ to compute the exponential smoothing values for the time-series. What is the forecast for week 7?

16.31 Admission application forms data (1000's) received by a management institute over the past 6 years are shown below:

Year :	1	2	3	4	5	6
Application forms :	20.5	20.2	19.5	19.0	19.1	18.8

Develop the equation for the linear trend component of this time-series. Comment on what is happening to admission forms for this institution.

16.32 Consider the following time-series data:

Quarter	*Year*		
	1	*2*	*3*
1	4	6	7
2	2	3	6
3	3	5	6
4	5	7	8

(a) Show the 4-quarter moving average values for this time-series.
(b) Compute seasonal indexes for the 4 quarters.

16.33 Below are given the figures of production (in million tonnes) of a cement factory:

Year	2003	2004	2005	2006	2007	2008	2009
Production	77	88	94	85	91	98	90

(a) Fit a straight line trend by the 'least squares method' and tabulate the trend values.
(b) Eliminate the trend. What components of the time-series are thus left over?
(c) What is the monthly increase in the production of cement? [*Sukhadia Univ., MBA, 2009*]

16.34 The sale of commodity in tonnes varied from January 2000 to December, 2000 in the following manner:

280	300	280	280	270	240
230	230	220	200	210	200

Fit a trend line by the method of semi-averages.

16.35 Fit a parabolic curve of the second degree to the data given below and estimate the value for 2002 and comment on it.

Year	2002	2003	2004	2005	2006
Sales (₹in '000)	10	12	13	10	8

16.36 Given below are the figures of production of a sugar (in 1000 quintals) factory:

Year	2001	2002	2003	2004	2005	2006	2007
Production	40	45	46	42	47	49	46

Fit a straight line trend by the method of least squares and estimate the value for 2009.

[*MBA, MD Univ., 2008*]

16.37 The following table gives the profits (₹in thousand) of a concern for 5 years ending 2000.

Year	2000	2001	2002	2003	2004
Profits	1.6	4.5	13.8	40.2	125.0

Fit an equation of the type $y = ab^x$.

Hints and Answers

16.25 (a) Forecasts is around 1200 (thousand) tonnes
(b) Averages are: 133, 300, 433, 500, 433, 466, 600 and 733.

16.26 (a) $\hat{y} = 489 + 75x$ [2005 = 0, x = years, y = tonnes in thousand]
(b) 11,64,000 tonnes

16.27 (a) 720 units when $x = 0$, 864 units when $x = 1$.
(b) $\hat{y} = 720 + 12x$ [July 1, 2003 = 0; x unit = 1 month;
y = annual sales rates in units]
720 units per year; 864 units per year.
(c) $\hat{y} = 60 + x$ [July 1, 2003 = 0, x unit = 1 month;
y = monthly sales rates in units]
60 units per month; 72 units per month.

16.28 (a) Longer average yield more smoothing; (b) 1.3

16.29 (a) $\hat{y} = 20 + 4x$ [1999 = 0, x = years; $\hat{y}$ = number of accidents];
(b) 44

16.30 (a)

Week (1)	*Values (2)*	*Forecast (3)* (4) = (2) – (3)	*Forecast Error*	*Squared Forecast*
1	8	—	—	—
2	13	—	—	—
3	15	—	—	—
4	17	12	5	25
5	16	15	1	1
6	9	16	–7	49

Forecast for week 7 is: (17 + 16 + 9)/3 = 14.

(b)

Week (t)	*Values* y_t	*Forecast* F_t $y_t - F_t$	*Forecast Error* $(y_t - F_t)^2$	*Squared Error*
1	8	—	—	—
2	13	8.00	5.00	25.00
3	15	9.00	6.00	36.00
4	17	10.20	6.80	46.24
5	16	11.56	4.44	19.71
6	9	12.45	–3.45	11.90
			138.85	

Forecast for week 7 is: 0.2 (9)+(1 – 0.2) (12.45) = 11.76.

16.31 $\sum x = 21, \sum x2 = 91, \sum y = 117.1, \sum xy = 403.7$

$$b = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} = \frac{6(403.7) - 21 \times 117.2}{6 \times 91 - (21)^2}$$

$= -0.3714$

$a = \bar{y} - b\bar{x} = 19.5167 - (-0.3514)(3.5) = 20.7466$

$\hat{y} = 20.7466 - 0.3514\,x.$

Enrolment appears to be decreasing by about 351 students per year.

16.32 (a)

Year	*Quarter* y	*Value*	*4-quarter Moving Average*	*Centred Moving Average*
1	1	4		
	2	2	3.50	
	3	3	4.00	3.750
	4	5	4.25	4.125
2	1	6	4.75	4.500
	2	3	5.25	5.000
	3	5	5.50	5.375
	4	7	6.25	5.875
3	1	7	6.50	6.375
	2	6	6.75	6.625
	3	6	—	
	4	8	—	

(b)

Year	*Quarter* y	*Value Moving Average*	*Centered Irregular Component*	*Seasonal*
1	1	4	—	
	2	2	—	
	3	3	3.750	0.8000
	4	5	4.125	1.2121
2	1	6	4.500	1.3333
	2	3	5.000	0.6000
	3	5	5.375	0.9302
	4	7	5.875	1.1915
3	1	7	6.375	1.0000
	2	6	6.625	0.9057
	3	6	—	—
	4	8	—	—

Quarter Component Values	*Seasonal-Irregular Index*	*Seasonal*
1	1.333, 1.0980	1.2157
2	0.6000, 0.9057	0.7529
3	0.8000, 0.9302	0.8651
4	1.2121, 1.1915	1.2018
		4.0355

$$\text{Adjusted for seasonal index} = \frac{4}{4.0355} = 0.9912.$$

16.33 (a)

Year	*Time Period*	*Production (in m. tonnes)* y	*Deviation From 1994* x	xy	x	*Trend Values* $\hat{y}$
2003	–4	77	–4	–308	16	83.299
2004	–2	88	–2	–176	4	86.051
2005	–1	94	–1	–94	1	87.427
2006	0	85	0	0	0	88.803
2007	1	91	1	91	1	90.179
2008	2	98	2	196	4	91.555
2009	5	90	5	450	25	95.683
		623	1	159	51	

Solving the normal equations

$$\sum y = na + b\sum x$$
$$623 = 7a + b$$
$$\sum xy = a\sum x + b\sum x2$$
$$159 = a + 5b$$

we get $a = 88.803$ and $b = 1.376x$. Thus

$$\hat{y} = a + bx = 88.803 + 1.376\,x$$

Substituting $x = -4, -2, -1, 0, 1, 2, 5$ to get trend values as shown above in the table.

(b) After eliminating the trend, we are left with S, C, and I components of time-series.

(c) Monthly increase in the production of cement in given by $b/12 = 1.376/12 = 0.115$.

16.34

Month	*Sales (in tonnes)*	
January	280	Total = 1650 of first six months;
February	300	
March	280	
April	280	Average = $\frac{1650}{6}$ = 275
May	270	
June	240	
July	230	Total = 1290 of last six months;
August	230	
September	220	
October	200	Average = $\frac{1290}{6}$ = 215.

Plot 275 and 215 in the middle of March-April 2000 and that of September-October 2000. By joining these two points we get a trend line which describes the given data.

16.35

Year	*Sales* y	*Period* x	xy	x^2	x^2y	x^4
2002	10	–2	–20	4	40	16
2003	12	–1	–12	1	12	1
2004	13	0	0	0	0	0
2005	10	1	10	1	10	1
2006	8	2	16	4	32	16
	53	0	–6	10	94	34

Parabolic trend line: $y = a + bx + bx2$

$$a = \frac{\sum y - c\sum x^2}{n} = \frac{53 - 0.857 \times 10}{5} = 8.886$$

$$b = \frac{\sum xy}{\sum x^2} = \frac{-6}{10} = -0.6;$$

$$c = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2} = \frac{5(94) - 10(53)}{5(34) - (10)^2} = -0.857$$

$y = 8.886 - 0.6x - 0.857x2$

For 2002, $x = 4$; $y = 8.886 - 0.6\ (4) - 0.857\ (4)2$

$= -7.226$

16.36

Year	*Production ('000 qtls)* y	*Deviations from 1994* x	xy	x^2
2001	40	−3	−120	9
2002	45	−2	−90	4
2003	46	−1	−46	1
2004	42	0	0	0
2005	47	1	47	1
2006	49	2	98	4
2007	46	3	138	9
	315	0	27	28

$\hat{y} = a + bx$; $a = \sum y/n = 315/7 = 45$;

$$b = \frac{\sum xy}{\sum x^2} = \frac{27}{28} = 0.964$$

$\hat{y} = 45 + 0.964x$

$= 45 + 0.964(9) = 45 + 8.676 = 53.676$

16.37

Year	*Profits* y	x	*Log y*	x^2	*x . Log y*
2000	1.6	−2	0.2041	4	–0.4082
2001	4.5	−1	0.6532	1	–0.6532
2002	13.8	0	1.1399	0	0
2003	40.2	1	1.6042	1	1.6042
2004	125.0	2	2.0969	4	4.1938
	185.1	0	5.6983	10	4.7366

Trend line: $y = ab^x$ or $\log y = \log a + x \log b$

where $\log a = \dfrac{\sum \log y}{n} = \dfrac{5.6983}{5} = 1.1397$;

$\log b = \dfrac{\sum x \log y}{\sum x^2} = \dfrac{4.7366}{10} = 0.474$

Thus $\log y = 1.1397 + 0.474\,x$.